Advanced Mathematics 2.1

Key Assignment #3

Name........................

Mystery Paper 1

1 Solve the inequality $|3x - 1| < |2x + 5|$. [4]

2 A curve is defined for $0 < \theta < \frac{1}{2}\pi$ by the parametric equations

$$x = \tan\theta, \quad y = 2\cos^2\theta\sin\theta.$$

Show that $\dfrac{dy}{dx} = 6\cos^5\theta - 4\cos^3\theta$. [5]

3 The polynomial $4x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ and $(x + 2)$ are factors of $p(x)$.

(i) Find the values of a and b. [4]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by $(x^2 + 1)$. [3]

4 **(i)** Show that $\cos(\theta - 60°) + \cos(\theta + 60°) \equiv \cos\theta$. [3]

(ii) Given that $\dfrac{\cos(2x - 60°) + \cos(2x + 60°)}{\cos(x - 60°) + \cos(x + 60°)} = 3$, find the exact value of $\cos x$. [4]

5 The complex numbers w and z are defined by $w = 5 + 3i$ and $z = 4 + i$.

(i) Express $\dfrac{iw}{z}$ in the form $x + iy$, showing all your working and giving the exact values of x and y. [3]

(ii) Find wz and hence, by considering arguments, show that

$$\tan^{-1}\left(\tfrac{3}{5}\right) + \tan^{-1}\left(\tfrac{1}{4}\right) = \tfrac{1}{4}\pi.$$ [4]

6 It is given that $I = \displaystyle\int_0^{0.3} \left(1 + 3x^2\right)^{-2} dx$.

(i) Use the trapezium rule with 3 intervals to find an approximation to I, giving the answer correct to 3 decimal places. [3]

(ii) For small values of x, $\left(1 + 3x^2\right)^{-2} \approx 1 + ax^2 + bx^4$. Find the values of the constants a and b. Hence, by evaluating $\displaystyle\int_0^{0.3} (1 + ax^2 + bx^4)\, dx$, find a second approximation to I, giving the answer correct to 3 decimal places. [5]

7 The equations of two straight lines are

$$\mathbf{r} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = a\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3a\mathbf{k}),$$

where a is a constant.

(i) Show that the lines intersect for all values of a. [4]

(ii) Given that the point of intersection is at a distance of 9 units from the origin, find the possible values of a. [4]

8 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \tfrac{1}{5}xy^{\frac{1}{2}}\sin\left(\tfrac{1}{3}x\right).$$

(i) Find the general solution, giving y in terms of x. [6]

(ii) Given that $y = 100$ when $x = 0$, find the value of y when $x = 25$. [3]

9 **(i)** Sketch the curve $y = \ln(x + 1)$ and hence, by sketching a second curve, show that the equation

$$x^3 + \ln(x + 1) = 40$$

has exactly one real root. State the equation of the second curve. [3]

(ii) Verify by calculation that the root lies between 3 and 4. [2]

(iii) Use the iterative formula

$$x_{n+1} = \sqrt[3]{(40 - \ln(x_n + 1))},$$

with a suitable starting value, to find the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(iv) Deduce the root of the equation

$$(e^y - 1)^3 + y = 40,$$

giving the answer correct to 2 decimal places. [2]

10 By first using the substitution $u = e^x$, show that

$$\int_0^{\ln 4} \frac{e^{2x}}{e^{2x} + 3e^x + 2}\,dx = \ln\left(\tfrac{8}{5}\right).$$ [10]

Mystery Paper 2

1 Find the exact value of the constant k for which $\int_1^k \frac{1}{2x-1}\,dx = 1$. [4]

2 The polynomial $x^4 + 3x^2 + a$, where a is a constant, is denoted by $p(x)$. It is given that $x^2 + x + 2$ is a factor of $p(x)$. Find the value of a and the other quadratic factor of $p(x)$. [4]

3 Use integration by parts to show that

$$\int_2^4 \ln x\,dx = 6\ln 2 - 2.$$ [4]

4 The curve with equation $y = e^{-x}\sin x$ has one stationary point for which $0 \leqslant x \leqslant \pi$.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

5 **(i)** Show that the equation

$$\tan(45^\circ + x) - \tan x = 2$$

can be written in the form

$$\tan^2 x + 2\tan x - 1 = 0.$$ [3]

(ii) Hence solve the equation

$$\tan(45^\circ + x) - \tan x = 2,$$

giving all solutions in the interval $0^\circ \leqslant x \leqslant 180^\circ$. [4]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$2 - x = \ln x$$

has only one root. [2]

(ii) Verify by calculation that this root lies between 1.4 and 1.7. [2]

(iii) Show that this root also satisfies the equation

$$x = \tfrac{1}{3}(4 + x - 2\ln x).$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \tfrac{1}{3}(4 + x_n - 2\ln x_n),$$

with initial value $x_1 = 1.5$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The number of insects in a population t days after the start of observations is denoted by N. The variation in the number of insects is modelled by a differential equation of the form

$$\frac{dN}{dt} = kN\cos(0.02t),$$

where k is a constant and N is taken to be a continuous variable. It is given that $N = 125$ when $t = 0$.

(i) Solve the differential equation, obtaining a relation between N, k and t. [5]

(ii) Given also that $N = 166$ when $t = 30$, find the value of k. [2]

(iii) Obtain an expression for N in terms of t, and find the least value of N predicted by this model. [3]

8 **(a)** The complex number z is given by $z = \dfrac{4 - 3i}{1 - 2i}$.

(i) Express z in the form $x + iy$, where x and y are real. [2]

(ii) Find the modulus and argument of z. [2]

(b) Find the two square roots of the complex number $5 - 12i$, giving your answers in the form $x + iy$, where x and y are real. [6]

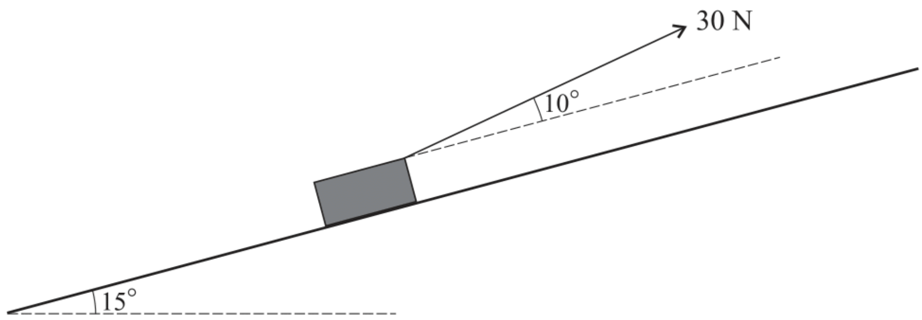
9 **(i)** Express $\dfrac{2 - x + 8x^2}{(1 - x)(1 + 2x)(2 + x)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{2 - x + 8x^2}{(1 - x)(1 + 2x)(2 + x)}$ in ascending powers of x, up to and including the term in x^2. [5]

There was a tenth question, but it had vector planes in it. This isn't part of the new syllabus so please draw a Llama instead. The llama is worth 12 marks and I will only award them for a quality llama. I am not joking. I never joke when it comes to llamas. They are a serious animal.

Mystery Paper 3

1



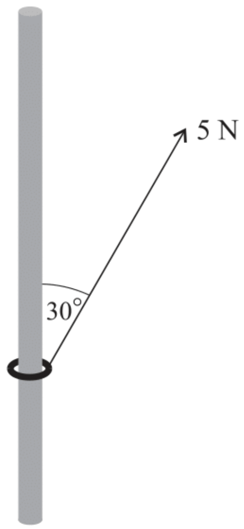
A box of mass 8 kg is pulled, at constant speed, up a straight path which is inclined at an angle of 15° to the horizontal. The pulling force is constant, of magnitude 30 N, and acts upwards at an angle of 10° from the path (see diagram). The box passes through the points A and B, where $AB = 20$ m and B is above the level of A. For the motion from A to B, find

(i) the work done by the pulling force, [2]

(ii) the gain in potential energy of the box, [2]

(iii) the work done against the resistance to motion of the box. [1]

2



A small ring of mass 0.6 kg is threaded on a rough rod which is fixed vertically. The ring is in equilibrium, acted on by a force of magnitude 5 N pulling upwards at 30° to the vertical (see diagram).

(i) Show that the frictional force acting on the ring has magnitude 1.67 N, correct to 3 significant figures. [2]

(ii) The ring is on the point of sliding down the rod. Find the coefficient of friction between the ring and the rod. [3]

3 A cyclist travels along a straight road working at a constant rate of 420 W. The total mass of the cyclist and her cycle is 75 kg. Ignoring any resistance to motion, find the acceleration of the cyclist at an instant when she is travelling at $5\ \text{m s}^{-1}$,

(i) given that the road is horizontal,

(ii) given instead that the road is inclined at $1.5°$ to the horizontal and the cyclist is travelling up the slope.

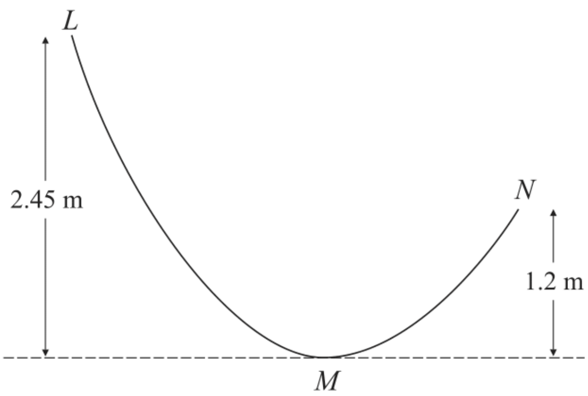
[5]

4 The velocity of a particle t s after it starts from rest is $v\ \text{m s}^{-1}$, where $v = 1.25t - 0.05t^2$. Find

(i) the initial acceleration of the particle, [2]

(ii) the displacement of the particle from its starting point at the instant when its acceleration is $0.05\ \text{m s}^{-2}$. [5]

5



The diagram shows the vertical cross-section LMN of a fixed smooth surface. M is the lowest point of the cross-section. L is 2.45 m above the level of M, and N is 1.2 m above the level of M. A particle of mass 0.5 kg is released from rest at L and moves on the surface until it leaves it at N. Find

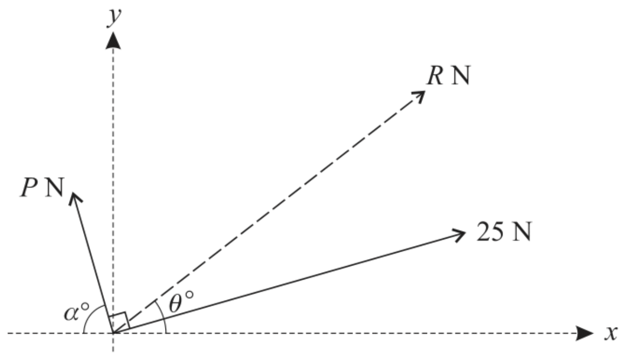
(i) the greatest speed of the particle, [3]

(ii) the kinetic energy of the particle at N. [2]

The particle is now projected from N, with speed $v\ \text{m s}^{-1}$, along the surface towards M.

(iii) Find the least value of v for which the particle will reach L. [2]

6



Forces of magnitudes P N and 25 N act at right angles to each other. The resultant of the two forces has magnitude R N and makes an angle of $\theta°$ with the x-axis (see diagram). The force of magnitude P N has components -2.8 N and 9.6 N in the x-direction and the y-direction respectively, and makes an angle of $\alpha°$ with the negative x-axis.

(i) Find the values of P and R. [3]

(ii) Find the value of α, and hence find the components of the force of magnitude 25 N in

(a) the x-direction,

(b) the y-direction.

[4]

(iii) Find the value of θ. [3]

7 A particle of mass m kg moves up a line of greatest slope of a rough plane inclined at $21°$ to the horizontal. The frictional and normal components of the contact force on the particle have magnitudes F N and R N respectively. The particle passes through the point P with speed $10\,\text{m s}^{-1}$, and 2 s later it reaches its highest point on the plane.

(i) Show that $R = 9.336m$ and $F = 1.416m$, each correct to 4 significant figures. [5]

(ii) Find the coefficient of friction between the particle and the plane. [1]

After the particle reaches its highest point it starts to move down the plane.

(iii) Find the speed with which the particle returns to P. [5]