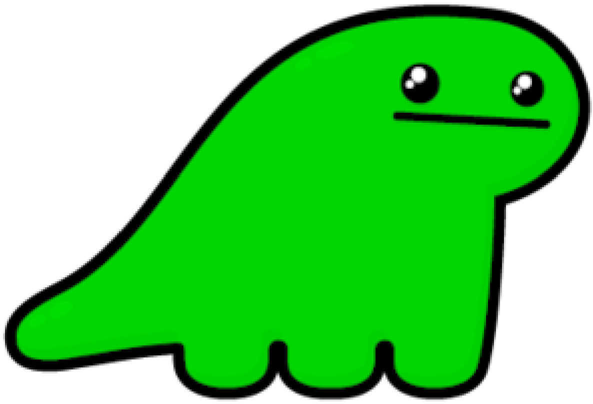
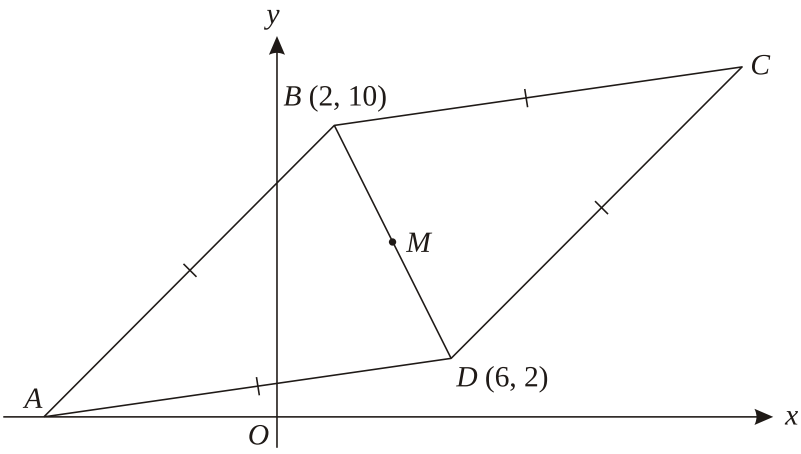
# PI



# COORDINATE GEOMETRY

1.




The diagram shows a rhombus *ABCD*. The points *B* and *D* have coordinates (2, 10) and (6, 2) respectively, and *A* lies on the *x*-axis. The mid-point of *BD* is *M*. Find, by calculation, the coordinates of each of *M*, *A* and *C*.

**[6]**

**2.** Three points have coordinates $A$ (2, 6), $B$ (8, 10) and $C$ (6, 0). The perpendicular bisector of $AB$ meets the line $BC$ at $D$. Find

(i) the equation of the perpendicular bisector of $AB$ in the form $ax + by = c$,

**[4]**

(ii) the coordinates of $D$.

**[4]**

**3.** The equation of a curve is $xy = 12$ and the equation of a line $l$ is $2x + y = k$, where $k$ is a constant.

(i) In the case where $k = 11$, find the coordinates of the points of intersection of $l$ and the curve. **[3]**

(ii) Find the set of values of $k$ for which $l$ does not intersect the curve. **[4]**

(iii) In the case where $k = 10$, one of the points of intersection is $P$ (2, 6). Find the angle, in degrees correct to 1 decimal place, between $l$ and the tangent to the curve at $P$. **[4]**

**4.** The curve $y^2 = 12x$ intersects the line $3y = 4x + 6$ at two points. Find the distance between the two points.

**[6]**

**5.** A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{\sqrt{(6-2x)}}$, and $P$ (1, 8) is a point on the curve.

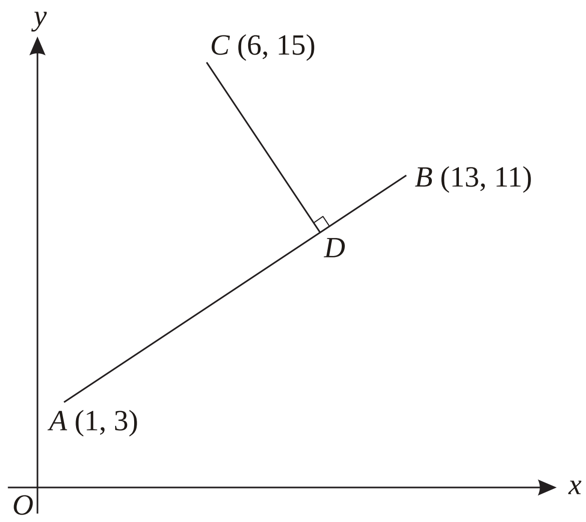
(i) The normal to the curve at the point $P$ meets the coordinate axes at $Q$ and at $R$. Find the coordinates of the mid-point of $QR$.

**[5]**

(ii) Find the equation of the curve.

**[4]**

**6.**




The three points $A$ (1, 3), $B$ (13, 11) and $C$ (6, 15) are shown in the diagram. The perpendicular from $C$ to $AB$ meets $AB$ at the point $D$. Find

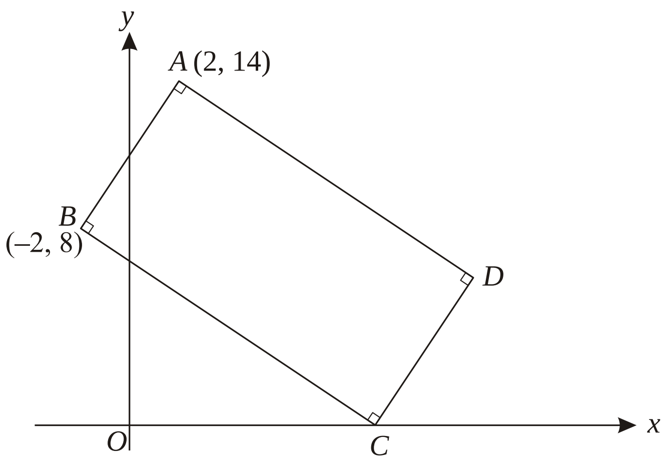
(i) the equation of $CD$,

**[3]**

(ii) the coordinates of $D$.

**[4]**

**7.**




The diagram shows a rectangle *ABCD*. The point *A* is (2, 14), *B* is (–2, 8) and *C* lies on the *x*-axis. Find

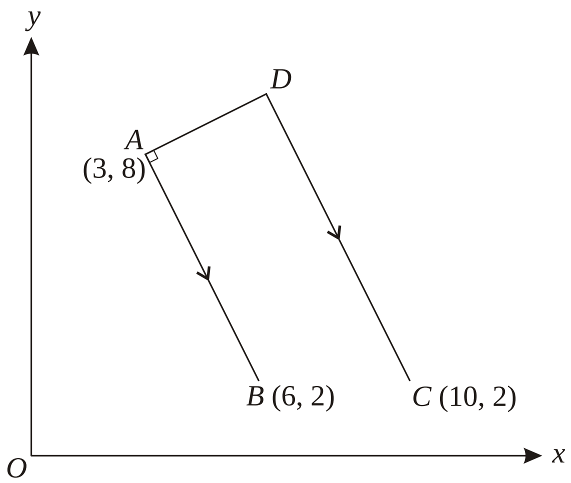
(i) the equation of *BC*,

**[4]**

(ii) the coordinates of *C* and *D*.

**[3]**

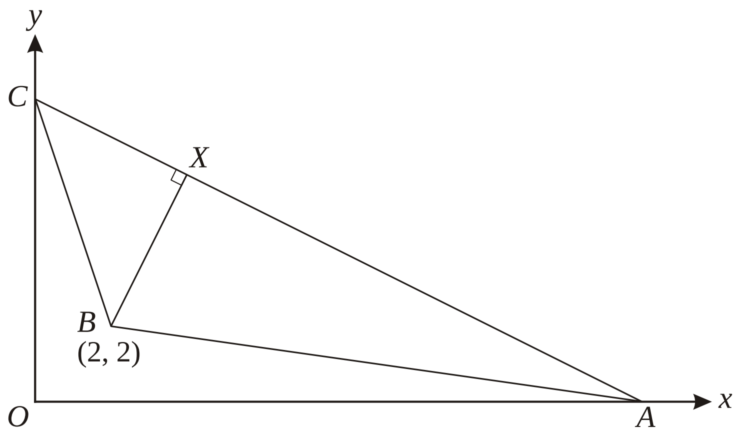
**8.**




The three points $A$ (3, 8), $B$ (6, 2) and $C$ (10, 2) are shown in the diagram. The point $D$ is such that the line $DA$ is perpendicular to $AB$ and $DC$ is parallel to $AB$. Calculate the coordinates of $D$.

**[7]**

**9.**




In the diagram, the points $A$ and $C$ lie on the $x$- and $y$-axes respectively and the equation of $AC$ is $2y + x = 16$. The point $B$ has coordinates (2, 2). The perpendicular from $B$ to $AC$ meets $AC$ at the point $X$.

(i) Find the coordinates of $X$.

**[4]**

The point $D$ is such that the quadrilateral $ABCD$ has $AC$ as a line of symmetry.

(ii) Find the coordinates of $D$.

**[2]**

(iii) Find, correct to 1 decimal place, the perimeter of $ABCD$.

**[3]**

**10.** The equation of a curve is $y = 5 - \dfrac{8}{x}$.

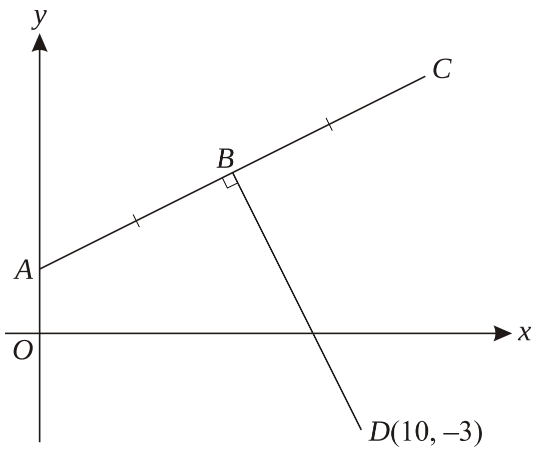
(i) Show that the equation of the normal to the curve at the point $P(2, 1)$ is $2y + x = 4$. **[4]**

This normal meets the curve again at the point $Q$.

(ii) Find the coordinates of $Q$. **[3]**

(iii) Find the length of $PQ$. **[2]**

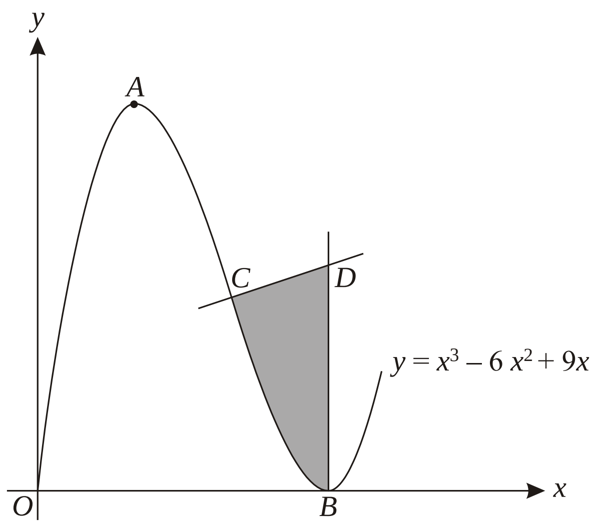
**11.**




The diagram shows points $A$, $B$ and $C$ lying on the line $2y = x + 4$. The point $A$ lies on the $y$-axis and $AB = BC$. The line from $D$ (10, –3) to $B$ is perpendicular to $AC$. Calculate the coordinates of $B$ and $C$.

**[7]**

**12.**




The diagram shows the curve $y = x^3 - 6x^2 + 9x$ for $x \geq 0$. The curve has a maximum point at $A$ and a minimum point on the $x$-axis at $B$. The normal to the curve at $C$ (2, 2) meets the normal to the curve at $B$ at the point $D$.

(i) Find the coordinates of $A$ and $B$.

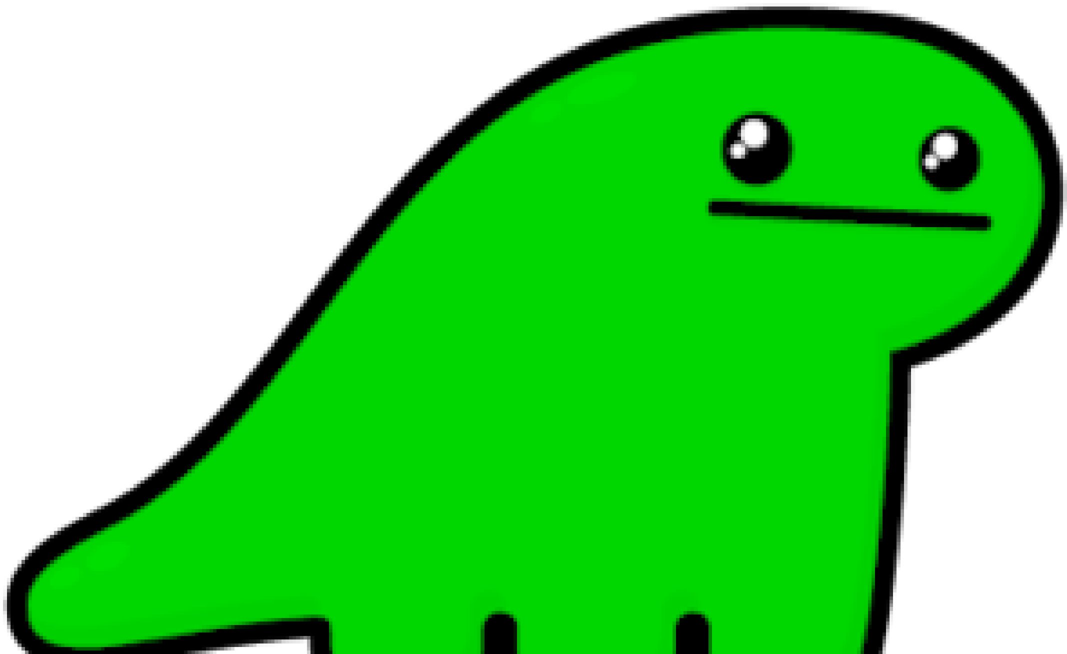
**[3]**

(ii) Find the equation of the normal to the curve at $C$.

**[3]**

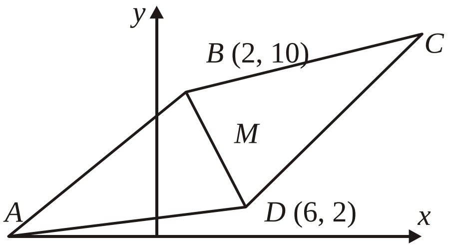
(iii) Find the area of the shaded region.

**[5]**



# SOLUTIONS

**1.**




| | |
|---|---|
| M (4, 6)<br>m of *BD* = –2<br>CAO | B1 |
| M of *AC* = $1/2$<br>Use of $m_1m_2 = -1$ | M1 |
| Eqn of *AC* $y - 6 = \frac{1}{2}(x - 4)$<br>Correct method leading to *A* – the equation may not be seen –$y = 0$ may be used with gradient. | M1 |
| → $x = -8$ when $y = 0$ A(–8, 0) | A1 |
| → $C = (16, 12)$ by vector move etc.<br>Any valid method - vectors, midpoint backwards, or solution of 2 sim eqns. | M1 A1 |

**[6]**

**2.** (i) *M* (5,8) B1
Co.

gradient of *AB* = $\frac{2}{3}$, Perp = $-\frac{3}{2}$ M1
Use of y step/x-step + $m_1m_2 = -1$ for *AB*

Equation $y - 8 = -\frac{3}{2}(x - 5)$ M1
Use of y – k = m(x – h) not (y + k) etc

→ $2y + 3x = 31$
(or locus method M1A1M1A1) A1 4

(ii) *BC*. $y = 5(x - 6)$ $y = 5x - 30$ M1A1
Use of y – k = m(x – h) not (y + k) etc. co

Sim Eqns → (7,5) DM1A1 4
Correct attempt at soln of *BC* with his answer to (i).

**3.** (i) $2x^2 + 12 = 11x$ or $y^2 - 11y + 24 = 0$ M1
Elimination of one variable completely

Solution → $(1\frac{1}{2}, 8)$ and $(4, 3)$ DM1
Correct method for soln of quadratic = 0

A1 3
co

Guesswork B1 for one, B3 for both.

(ii) $2x^2 - kx + 12 = 0$ M1
Used on quadratic=0. Allow = 0, >0 etc

Use of $b^2 - 4ac$ A1
For $k^2 - 96$

$k^2 < 96$ DM1
Definite use of $b^2 - 4ac < 0$

$-\sqrt{96} < k < \sqrt{96}$ or $|k| < \sqrt{96}$ A1 4
Co. (condone inclusion of ≤)

(iii) gradient of $2x + y = k = -2$ B1
Anywhere

$dy/dx = -12/x^2$ $(= -3)$ B1
For differentiation only – unsimplified

Use of tangent for an angle M1
Used with either line or tangent

Difference = 8.1° or 8.2° A1 4
Co

**[11]**

**4.** $y^2 = 12x$ and $3y = 4x + 6$
Complete elimination of 1 variable. M1
$x$ or $y$ must be removed completely.

→ $y^2 - 9y + 18 = 0$ or $4x^2 - 15x + 9 = 0$ A1
Must be a 3 term quad – not nec = 0.

Solution → $(\frac{3}{4}, 3)$ and $(3,6)$ DM1 A1
Correct method of solution. co.

Distance = $\sqrt{(3^2 + 2.25^2)}$ = 3.75 M1 A1
Correct method including √. co.

**[6]**

**5.** (i) $\frac{dy}{dx} = \frac{4}{\sqrt{6-2x}}$.

if $x = 1$, $m = 2$ and perp $m = -\frac{1}{2}$ — M1 A1

Use of $m_1m_2 = -1$. A1 co for $-\frac{1}{2}$

$\rightarrow y - 8 = -\frac{1}{2}(x - 1)$ $(2y + x = 17)$ — DM1

Any correct form of perpendicular.

$\rightarrow (0, 8\frac{1}{2})$ and $(17,0)$ — A1

co.

$\rightarrow M(8\frac{1}{2}, 4\frac{1}{4})$ — B1√ — 5

For his answers.

(ii) $y = \frac{4(6-2x)^{\frac{1}{2}}}{\frac{1}{2}\times -2}$ + c — B1

For 4, $(6 - 2x)^{\frac{1}{2}}$ and + $\frac{1}{2}$ and no other f($x$)

M1

For ÷ –2 (only if no other f($x$))

→ subs (1, 8) → c = 16 — M1A1 — 4

Substituting into any integrated expression to find c.

**[9]**

**6.** (i) m of AB = 8/12

m of perpendicular = –12/8 — M1

Use of $m_1m_2 = -1$ and $y$-step/$x$-step

eqn of $CD$ $y - 15 = -\frac{3}{2}(x-6)$ — M1 A1 — 3

Correct form of eqn of line. co.

(ii) eqn of $AB$ $y - 3 = \frac{2}{3}(x-1)$ — M1 A1√

Could be given in (i)

Sim eqns $2y + 3x = 48$ and $3y = 2x + 7$ — DM1

Needs both M marks from (i)

$\rightarrow D\ (10, 9)$ — A1 — 4

co.

**[7]**

**7.** (i) m of $AB$ = 1.5 (or $1\frac{1}{2}$) B1

co anywhere

m of $BC = -1 \div$ (m of $AB$) $= -\frac{2}{3}$ M1

Use of $m_1m_2 = -1$

$\to$ Eqn $y - 8 = -\frac{2}{3}(x + 2)$ or $3y + 2x = 20$ M1 A1√ 4

Correct form used – or $y = mx + c$. co

(√ needs both M marks)

(ii) Put $y = 0 \to C\ (10, 0)$ B1√

√ in his linear equation.

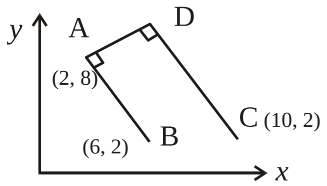
Vector move $\to D\ (14, 6)$ M1A1 3

completely correct method. Co

(or sim eqns $3y + 2x = 46$ and $2y = 3x - 30$)

**[7]**

**8.**




Gradient of $AB = -2$ B1

co

Eqn of $CD$ $y - 2 = -2(x - 10)$

($y + 2x = 22$) M1 A1√

correct form of eqn (inc $y = mx + c$)

awarded for either $CD$ or $AD$.

accept any form for A mark.

Uses $m_1m_2 = -1$ M1

Use of $m_1m_2 = -1$

Eqn of DA $y - 8 = \frac{1}{2}(x - 3)$

($2y = x + 13$) A1√

Any correct form.

Sim eqns $\to$ (6.2, 9.6) M1A1

Reasonable algebra. co

**[7]**

**9.** (i) Gradient of $AC = -\frac{1}{2}$ B1
Correct gradient.

Perpendicular gradient = 2 M1
Use of $m_1m_2 = -1$

Eqn of $BX$ is $y - 2 = 2(x - 2)$ M1
Correct form of equation

Sim Eqns $2y + x = 16$ with $y = 2x - 2$
$\rightarrow (4, 6)$ A1 4
co

(ii) $X$ is mid-point of $BD$, $D$ is (6, 10) M1 A1√ 2
Any valid method. ft on (i).

(iii) $AB = \sqrt{(14^2 + 2^2)} = \sqrt{200}$
$BC = \sqrt{(2^2 + 6^2)} = \sqrt{40}$ M1
Use of Pythagoras once.

$\rightarrow$ Perimeter = $2\sqrt{200} + 2\sqrt{40}$ DM1
4 lengths added.

$\rightarrow$ Perimeter = 40.9 A1 3
co

**[9]**

**10.** $y = 5 - \frac{8}{x}$, $P(2, 1)$

(i) $\frac{dy}{dx} = \frac{8}{x^2}$ B1
Correct differentiation

$m$ of tan = $2m$ of normal $= -\frac{1}{2}$ M1
Use of $m_1m_2 = -1$

Eqn of normal $y - 1 = -\frac{1}{2}(x - 2)$ M1
Correct method for line

$\rightarrow 2y + x = 4$ A1 4
Answer given

| | | | | |
|---|---|---|---|---|
| | (ii) | Sim eqns $2y + x = 4$, $y = 5 - \frac{8}{x}$ | | |
| | | $\rightarrow x^2 + 6x - 16 = 0$ or $y^2 - 7y + 6 = 0$<br>Complete elimination of $x$ or $y$ | M1 | |
| | | $\rightarrow (-8, 6)$<br>Soln of quadratic. Co | DM1 A1 | 3 |
| | (iii) | Length = $\sqrt{10^2 + 5^2} = \sqrt{125}$<br>Correct use of Pythagoras | M1 | |
| | | $\rightarrow 11.2$ (accept $\sqrt{125}$ or $5\sqrt{5}$ etc)<br>For his points. | A1 | 2 |
| | | | **[9]** | |
| **11.** | | $m$ of $AC = \frac{1}{2}$ | | |
| | | Perpendicular gradient = –2<br>Use of $m_1m_2 = -1$ | M1 | |
| | | Eqn $BD$ $y + 3 = -2(x - 10)$<br>Correct method for eqn of line | M1 | |
| | | ( or $y + 2x = 17$)<br>In any form. | A1 | |
| | | Sim. eqns $BD$ with given eqn.<br>Correct method of solution. | M1 | |
| | | $\rightarrow B$ (6, 5)<br>co. | A1 | |
| | | Vector move (step) $\rightarrow C$ (12, 8)<br>Any valid method. √ for his $B$. | M1 A1√ | |
| | | | **[7]** | |
| **12.** | (i) | $\frac{dy}{dx} = 3x^2 - 12x + 9$<br>co (can be given in part (ii)) | B1 | |
| | | Solves $\frac{dy}{dx} = 0$<br>Attempt to solve d$y$/d$x$ = 0. | M1 | |
| | | $\rightarrow$ A (1, 4), $B$ (3, 0).<br>Both needed. | A1 | 3 |

(ii) If $x = 2$, $m = –3$

Normal has $m = \frac{1}{3}$ M1

Use of $m_1m_2 = –1$. needs calculus.

Eqn $y – 2 = \frac{1}{3}(x – 2)$ or $3y = x + 4$. M1 A1 3

Correct form of equation – needs calculus.
A1 any form.

(iii) area under curve – integrate $y$.

$\rightarrow \frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2$ B2,1

For the 3 terms. –1 for each error.

Limits 2 to "his 3" $\rightarrow \frac{3}{4}$ (0.75) M1

Using 2 to "his 3" with integration.

Area of trapezium = $\frac{1}{2} \times 1 \times (2 + 2\frac{1}{3})$

$= 2\frac{1}{6}$ M1

Any correct method for trapezium.

Subtract $\rightarrow$ shaded area of $1\frac{5}{12}$ A1 5

co

**[11]**