PI

EQUATIONS AND INEQUALITIES

1. Solve the simultaneous equations

$$x - 2y = 1,$$

$$x^2 + y^2 = 29.$$

(Total 6 marks)

2. Solve the simultaneous equations

$$x + y = 2$$

$$x^2 + 2y = 12.$$

(Total 6 marks)

3. Solve the simultaneous equations

$$x + y = 3,$$

$$x^2 + y = 15.$$

(Total 6 marks)

4. Find the set of values of x for which

$$(2x + 1)(x - 2) > 2(x + 5).$$

(Total 7 marks)

5. Find the set of values for x for which

(a) $6x - 7 < 2x + 3$,

(2)

(b) $2x^2 - 11x + 5 < 0$,

(4)

(c) both $6x - 7 < 2x + 3$ and $2x^2 - 11x + 5 < 0$.

(1)

(Total 7 marks)

6. Solve the inequality

$$10 + x^2 > x(x - 2) .$$

(Total 3 marks)

7. Draw a picture of a giraffe flying a biplane

(Total 3 marks)

8. (a) Show that eliminating y from the equations

$2x + y = 8,$

$3x^2 + xy = 1$

produces the equation

$x^2 + 8x - 1 = 0.$

(2)

(b) Hence solve the simultaneous equations

$2x + y = 8,$

$3x^2 + xy = 1$

giving your answers in the form $a + b\sqrt{17}$, where a and b are integers.

(5)

(Total 7 marks)

9. (a) Given that $3^x = 9^{y-1}$, show that $x = 2y - 2$.

(2)

(b) Solve the simultaneous equations

$$x = 2y - 2,$$

$$x^2 = y^2 + 7.$$

(6)
(Total 8 marks)

10. **Solve the simultaneous equations**

$$x - 3y + 1 = 0,$$

$$x^2 - 3xy + y^2 = 11.$$

(Total 7 marks)

11. The curve C has equation $y = x^2 - 4$ and the straight line l has equation $y + 3x = 0$.

(a) In the space below, sketch C and l on the same axes.

3)

(b) Write down the coordinates of the points at which C meets the coordinate axes.

(2)

(c) Using algebra, find the coordinates of the points at which l intersects C.

(4)
(Total 9 marks)

12. Find the set of values of x for which

(a) $3(2x + 1) > 5 - 2x$,

(2)

(b) $2x^2 - 7x + 3 > 0$,

(4)

(c) **both** $3(2x + 1) > 5 - 2x$ **and** $2x^2 - 7x + 3 > 0$.

(2)
(Total 8 marks)

SOLUTIONS

1.	$x = 1 + 2y$ and sub $\rightarrow (1 + 2y)^2 + y^2 = 29$	M1
	$\Rightarrow 5y^2 + 4y - 28 (= 0)$	A1
	i.e. $(5y + 14)(y - 2) = 0$	M1
	$(y =)2$ or $-\frac{14}{5}$ (o.e.)	(both) A1
	$y = 2, \Rightarrow x = 1 + 4 = 5$; $y = -\frac{14}{5} \Rightarrow x = -\frac{23}{5}$ (o.e)	M1
		[6]
2.	$x^2 + 2(2 - x) = 12$ or $(2 - y)^2 + 2y = 12$ (Eqn. in x or y only)	M1
	$x^2 - 2x - 8 = 0$ or $y^2 - 2y - 8 = 0$ (Correct 3 term version) *(Allow, e.g. $x^2 - 2x = 8$)*	A1
	$(x - 4)(x + 2) = 0$ $x = \dots$ or $(y - 4)(y + 2) = 0$ $y = \dots$	M1
	$x = 4$, $x = -2$ or $y = 4$, $y = -2$	A1
	$y = -2$, $y = 4$ or $x = -2$, $x = 4$ (M: attempt one, A: both)	M1 A1ft
		[6]
3.	Forming equation in x or y by attempt to eliminate one variable	
	$(3 - y)^2 + y = 15$ *or* $x^2 + (3 - x) = 15$	M1
	$y^2 - 5y - 6 = 0$ *or* $x^2 - x - 12 = 0$ (Correct 3 term version)	A1
	Attempt at solution i.e. solving 3 term quadratic: $(y - 6)(y + 1) = 0$, $y = \dots$ or $(x - 4)(x + 3) = 0$, $x = \dots$ or correct use of formula or	M1
	correct use of completing the square	A1
	$x = 4$ and $x = -3$ **or** $y = -1$ and $y = 6$	M1 A1 ft
	Finding the values of the other coordinates (M attempt one, A both)	
		[6]
4.	$(2x + 1)(x - 2) > 2(x + 5)$	
	$2x^2 - 4x + x - 2 > 2x + 10$	M1 A1
	$2x^2 - 5x - 12 > 0$	A1 ft
	$(2x + 3)(x - 4) > 0$ (or solving M1 A1)	M1 A1 ft
	$x < -\frac{3}{2}$, $x > 4$	M1 A1 7
		[7]
5.	(a) $6x - 2x < 3 + 7$ $x < 2\frac{1}{2}$	M1 A1 2
	(b) $(2x - 1)(x - 5)$ Critical values $\frac{1}{2}$ and 5	M1 A1
	$\frac{1}{2} < x < 5$	M1 A1 ft
	(c) $\frac{1}{2} < x < 2\frac{1}{2}$	B1 ft 1

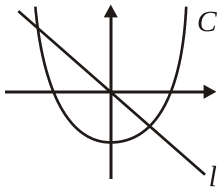
6.	$10 + x^2 > x^2 - 2x$	B1	
	$10 > -2x \qquad x > -5$	M1 A1	3
		[3]	

8.	(a)	$y = 8 - 2x \qquad 3x^2 + x(8 - 2x) = 1$ M1 $x^2 + 8x - 1 = 0$ **(*)**	A1	2
	(b)	$x = \dfrac{-8 \pm \sqrt{64+4}}{2} = -4 \pm \ldots$	M1 A1	
		$\sqrt{68} = 2\sqrt{17}$; $x = -4 + 2\sqrt{17}$ or $x = -4 - 2\sqrt{17}$	B1	
		$y = 8 - 2(-4 + \sqrt{17}) = 16 - 2\sqrt{17}$ or $y = 16 + 2\sqrt{17}$	M1 A1	5
			[7]	

9.	(a)	$3^x = 3^{2(y-1)} \qquad x = 2(y-1)$ (*)	M1 A1
	(b)	$(2y - 2)^2 = y^2 + 7, \qquad 3y^2 - 8y - 3 = 0$	M1, A1
		$(3y + 1)(y - 3) = 0$, $y = \ldots$ (or correct substitution in formula)	M1
		$y = -\dfrac{1}{3}, \qquad y = 3$	A1
		$x = -\dfrac{8}{3}, \qquad x = 4$	M1 A1 ft
			[8]

10.	$x = 3y - 1$ (n.b. Method mark, so allow, e.g. $x = 3y + 1$)	M1
	$(3y - 1)^2 - 3y(3y - 1) + y^2 = 11$ (Substitution, leading to an equation in only one variable)	M1
	$y^2 - 3y - 10 = 0$ (3 terms correct, "=0" possibly implied)	A1
	$(y - 5)(y + 2) = 0 \qquad y = 5 \qquad y = -2$	M1 A1
	$x = 14 \qquad x = -7$	M1 A1 ft
	(If not exact, f.t. requires at least 1 d.p. accuracy).	
	Alternative approach gives: $y = \dfrac{x+1}{3}$, $x^2 - 7x - 98 = 0$.	
		[7]

11. (a)



C : "U" shape — B1

C : Position — B1

l : Straight line through origin with negative gradient — B1 — 3

(b) (2, 0), (–2, 0), (0, – 4)

2 of these correct: — B1

All 3 correct: — B1 — 2

(c) $x^2 - 4 = -3x$

$x^2 + 3x - 4 = 0 \quad (x + 4)(x - 1) = 0 \quad x = \ldots$ — M1

$x = -4 \qquad x = 1$ — A1

$y = 12 \qquad y = -3$ M: Attempt one y value — M1 A1 — 4

[9]

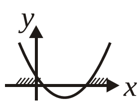
12. (a) $6x + 3 > 5 - 2x \Rightarrow 8x > 2$ — M1

$x > \frac{1}{4}$ or 0.25 or $\frac{2}{8}$ — A1 — 2

M1 Multiply out and collect terms (allow one slip and allow use of = here)

(b) $(2x - 1)(x - 3)$ (> 0) — M1

Critical values $x = \frac{1}{2}$, 3 — (both) A1



Choosing "outside" region — M1

$x > 3$ or $x < \frac{1}{2}$ — A1 ft — 4

1st M1 Attempting to factorise 3TQ $\rightarrow x = \ldots$

2nd M1 Choosing the outside region

2nd A1 f.t. f.t. their critical values N.B.($x > 3$, $x > \frac{1}{2}$ is M0A0)

For $p < x < q$ where $p > q$ penalise the final A1 in (b)

(c) $x > 3$ or $\frac{1}{4} < x < \frac{1}{2}$ — B1f.t. B1f.t.

f.t. their answers to (a) and (b)

1st B1 a correct f.t. leading to an infinite region

2nd B1 a correct f.t. leading to a finite region

Penalise $\leq$ or $\geq$ once only at first offence.

e.g.	*(a)*	*(b)*	*(c)*	*Mark*
	$x > \frac{1}{4}$	$\frac{1}{2} < x < 3$	$\frac{1}{2} < x < 3$	*B0 B1*
	$x > \frac{1}{4}$	$x > 3, x > \frac{1}{2}$	$x > 3$	*B1 B0*