PI

QUADRATICS

1. $$x^2 - 8x - 29 \equiv (x + a)^2 + b,$$

where a and b are constants.

(a) Find the value of a and the value of b.

(3)

(b) Hence, or otherwise, show that the roots of

$$x^2 - 8x - 29 = 0$$

are $c \pm d\sqrt{5}$, where c and d are integers to be found.

(3)

(Total 6 marks)

2. (a) Solve the equation $4x^2 + 12x = 0$.

(3)

$$f(x) = 4x^2 + 12x + c,$$

where c is a constant.

(b) Given that $f(x) = 0$ has equal roots, find the value of c and hence solve $f(x) = 0$.

(4)

(Total 7 marks)

3. $f(x) = x^2 - kx + 9$, where k is a constant.

(a) Find the set of values of k for which the equation $f(x) = 0$ has no real solutions.

(4)

Given that $k = 4$,

(b) express $f(x)$ in the form $(x - p)^2 + q$, where p and q are constants to be found,

(3)

(c) write down the minimum value of $f(x)$ and the value of x for which this occurs.

(2)

(Total 9 marks)

4. The function f is even and has domain $\mathbb{R}$. For $x \geq 0$, $f(x) = x^2 - 4ax$, where a is a positive constant.

(a) In the space below, sketch the curve with equation $y = f(x)$, showing the coordinates of all the points at which the curve meets the axes.

(3)

(b) Find, in terms of a, the value of $f(2a)$ and the value of $f(-2a)$.

(2)

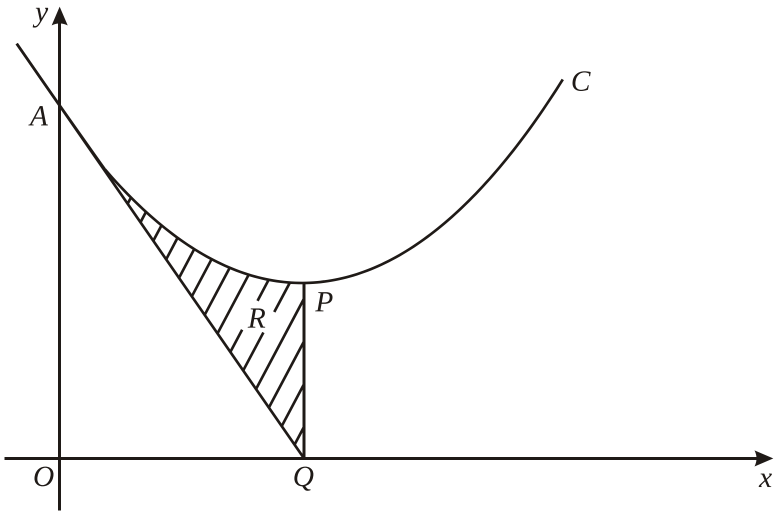
Given that $a = 3$,

(c) use algebra to find the values of x for which $f(x) = 45$.

(4)

(Total 9 marks)

5.



The diagram above shows part of the curve C with equation $y = x^2 - 6x + 18$. The curve meets the y-axis at the point A and has a minimum at the point P.

(a) Express $x^2 - 6x + 18$ in the form $(x - a)^2 + b$, where a and b are integers.

(3)

(b) Find the coordinates of P.

(2)

(c) Find an equation of the tangent to C at A.

(4)

The tangent to C at A meets the x-axis at the point Q.

(d) Verify that PQ is parallel to the y-axis.

(1)

The shaded region R in the diagram is enclosed by C, the tangent at A and the line PQ.

(e) Use calculus to find the area of R.

(5)

(Total 15 marks)

6. Given that the equation $kx^2 + 12x + k = 0$, where k is a positive constant, has equal roots, find the value of k.

(Total 4 marks)

7. Given that

$$f(x) = x^2 - 6x + 18, \quad x \geq 0,$$

(a) express f(x) in the form $(x - a)^2 + b$, where a and b are integers.

(3)

The curve C with equation $y = f(x)$, $x \geq 0$, meets the y-axis at P and has a minimum point at Q.

(b) Sketch the graph of C, showing the coordinates of P and Q.

(4)

The line $y = 41$ meets C at the point R.

(c) Find the x-coordinate of R, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers.

(5)

(Total 12 marks)

SOLUTIONS

1. (a) $a = -4$ B1

$x^2 - 8x - 29 \equiv (x \pm 4)^2 - 16(-29)$, $b = -45$ M1A1 3

B1 is for $(x-4)^2$ or $a = -4$

M1 requires $(x \pm p)^2 - p^2(\pm 29)$, $p \neq 0$,

A1 is for $b = -45$

Answer only: full marks.

Note: Can score B0M1A1 [e.g. $(x+4)^2 - 45$]

Comparing coefficients: M1 is for comparing x coefficients and constant term

(b) $x - 4 = (\pm\sqrt{45})$, $[\sqrt{45} = 3\sqrt{5}]$ M1

$x = 4 \pm 3\sqrt{5}$ cao $c = 4$, $d = 3$ A1 A1 3

Or: $x = \dfrac{8 \pm \sqrt{64 + 116}}{2}$, $\sqrt{180} = 6\sqrt{5}$ M1 A1

[6]

2. (a) $4x(x + 3) = 0$, $x = \ldots$ (or use of quadratic formula) M1

$x = 0$ $x = -3$ A1 A1 3

(b) Using $b^2 - 4ac = 0$ or other method, proceed to $c = \ldots$ M1

$c = 9$ A1

$(2x + 3)(2x + 3) = 0$ $x = \ldots$ (or other method to solve a 3-term quadratic) M1

$x = -\dfrac{3}{2}$ A1 4

[7]

3. (a) $b^2 - 4ac = (-k)^2 - 36 = k^2 - 36$ M1 A1

Or, (completing the square), $\left(x - \dfrac{1}{2}k\right)^2 = \dfrac{1}{4}k^2 - 9$

Or, if b^2 and $4ac$ are compared directly, [M1] for finding both [A1] for k^2 and 36. M1 A1

No real solutions: $k^2 - 36 < 0$, $-6 < k < 6$ (ft their "36") M1, A1ft

(b) $x^2 - 4x + 9 = (x - 2)^2$ ………. ($p = 2$) B1

Ignore statement $p = -2$ if otherwise correct.

$x^2 - 4x + 9 = (x - 2)^2 - 4 + 9 = (x - 2)^2 + 5$ ($q = 5$) M1 A1 3

M: Attempting $(x \pm a)^2 \pm b \pm 9$, $a \neq 0$, $b \neq 0$.

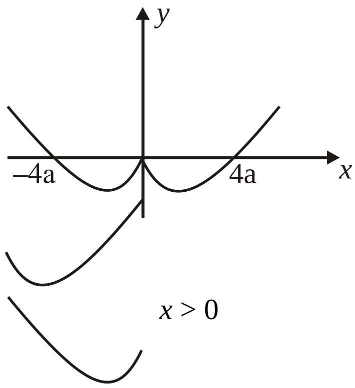
(c) Min value 5 (or just q), occurs where $x = 2$ (or just p) B1ft, B1ft

<u>Alternative:</u> $f'(x) = 2x - 4$ (Min occurs where) $x = 2$ [B1]

Where $x = 2$, $f(x) = 5$ [B1ft]

[9]

4. (a)



$x > 0$ B1

$x < 0$ B1 ft

$(4a, 0)$ & $(–4a, 0)$ and shape at $(0, 0)$ B1 3

(b) $f(2a) = (2a)^2 – 4a(2a) = 4a^2 – 8a^2 = \underline{–4a^2}$ B1

$f(–2a)$ [= $f(2a)$ ($\because$ *even function*)] = $\underline{–4a^2}$ B1 ft 2

B1 ft their f(2a)

(c) $a = 3$ and $f(x) = 45 \Rightarrow 45 = x^2 – 12x$ $(x > 0)$ M1

$0 = x^2 – 12x – 45$

$0 = (x – 15)(x + 3)$ M1

$x = 15$ (or $–3$) A1

$\therefore$ Solutions are $\underline{x = \pm 15}$ <u>only</u> A1 4

M1 Attempt 3TQ in x

M1 Attempt to solve

A1 At least x = 15 can ignore x = –3

A1 To get final A1 must make clear <u>*only*</u> *answers are ±15.*

[9]

5. (a) $(x – 3)^2$, +9 isw . $a = 3$ and $b = 9$ may just be written down with no method shown. B1, M1 A1

(b) P is (3, 9) B1

(c) A = (0, 18) B1

$\frac{dy}{dx} = 2x – 6$, at A $m = –6$ M1 A1

Equation of tangent is $y – 18 = –6x$ (in any form) A1ft 4

(d) Showing that line meets x axis directly below P, i.e. at $x = 3$. A1cso 1

(e) $A = \int x^2 – 6x + 18x = [\frac{1}{3}x^3 – 3x^2 + 18x]$ M1 A1

Substituting $x = 3$ to find area A under curve A [=36] M1

Area of $R = A$ – area of triangle = $A – \frac{1}{2} \times 18$ 3, = 9 M1 A1 5

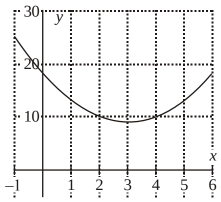
Alternative: $\int x^2 – 6x + 18 – (18 – 6x)dx$ M1

$= \frac{1}{3}x^3$ M1 A1 ft

Use $x = 3$ to give answer 9 M1 A1

[13]

6.	Attempt to use discriminant $b^2 - 4ac$	Should have no x's	M1	
	(Need not be equated to zero)	(Could be within the quadratic formula)		
	$144 - 4 \times k \times k = 0$ or $\sqrt{144 - 4 \times k \times k} = 0$		A1	
	Attempt to solve for k	(Could be an inequality)	M1	
	$k = 6$		A1	4
			[4]	

7.	(a)	$x^2 - 6x + 18 = (x - 3)^2, +9$		B1, M1 A1	
	(b)				
		"U"-shaped parabola		M1	
		Vertex in correct quadrant		A1ft	
		P: (0, 18) (or 18 on y-axis)		B1	
		Q: (3, 9)		B1ft	4
	(c)	$x^2 - 6x + 18 = 41$ or $(x - 3)^2 + 9 = 41$		M1	
		Attempt to solve 3 term quadratic $x = \ldots$		M1	
		$x = \frac{6 \pm \sqrt{36 - (4 \times -23)}}{2}$	(or equiv.)	A1	
		$\sqrt{128} = \sqrt{64} \times \sqrt{2}$	(or surd manipulation $\sqrt{2a} = \sqrt{2}\sqrt{a}$)	M1	
		$3 + 4\sqrt{2}$		A1	5
				[12]	