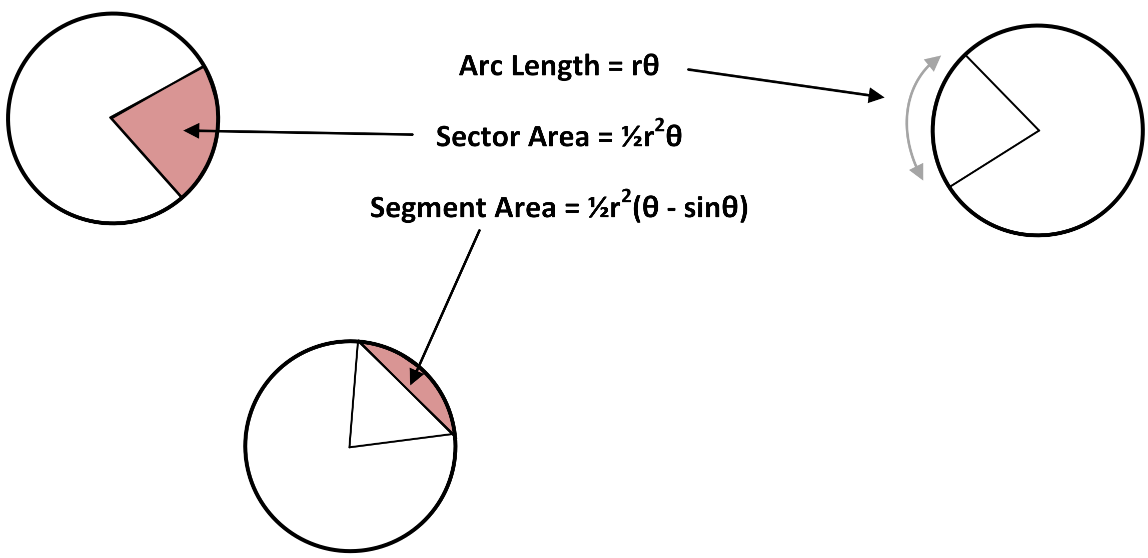
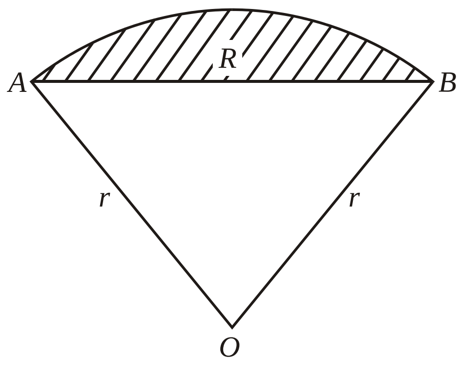
PI

BITS OF CIRCLES



1.



The diagram above shows the sector *OAB* of a circle of radius *r* cm. The area of the sector is 15 cm^2 and $\angle AOB$ = 1.5 radians.

(a) Prove that $r = 2\sqrt{5}$.

(3)

(b) Find, in cm, the perimeter of the sector *OAB*.

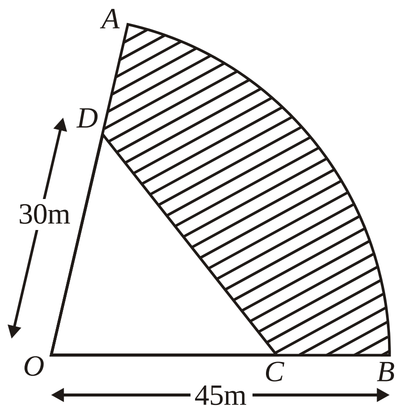
(2)

The segment *R*, shaded in the diagram above, is enclosed by the arc *AB* and the straight line *AB*.

(c) Calculate, to 3 decimal places, the area of *R*.

(3)
(Total 8 marks)

2.



A fence from a point *A* to a point *B* is in the shape of an arc *AB* of a circle with centre *O* and radius 45 m, as shown in the diagram. The length of the fence is 63 m.

(a) Show that the size of $\angle AOB$ is exactly 1.4 radians.

(2)

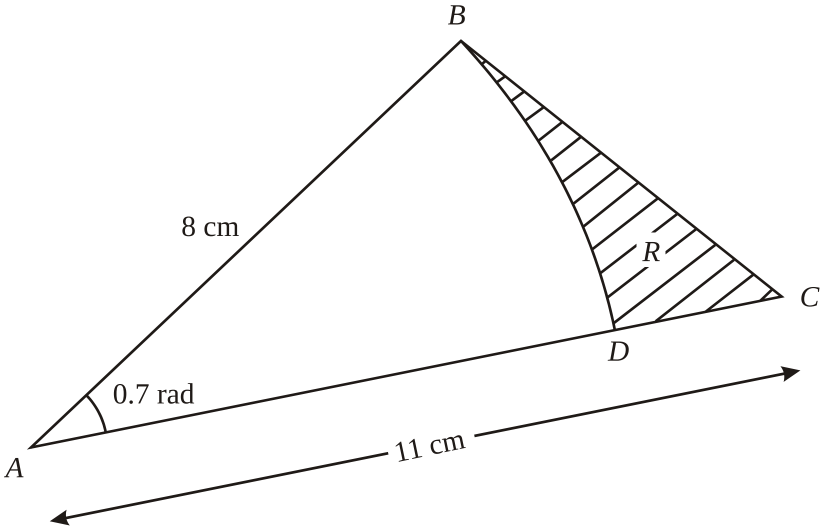
The points *C* and *D* are on the lines *OB* and *OA* respectively, with *OC* = *OD* = 30 m.

A plot of land *ABCD*, shown shaded in the figure above, is enclosed by the arc *AB* and the straight lines *BC*, *CD* and *DA*.

(b) Calculate, to the nearest m^2, the area of this plot of land.

(5)
(Total 7 marks)

3.



This diagram shows the triangle *ABC*, with *AB* = 8 cm, *AC* = 11 cm and ∠ *BAC* = 0.7 radians. The arc *BD*, where *D* lies on *AC*, is an arc of a circle with centre *A* and radius 8 cm. The region *R*, shown shaded in the diagram, is bounded by the straight lines *BC* and *CD* and the arc *BD*.

Find

(a) the length of the arc *BD*,

(2)

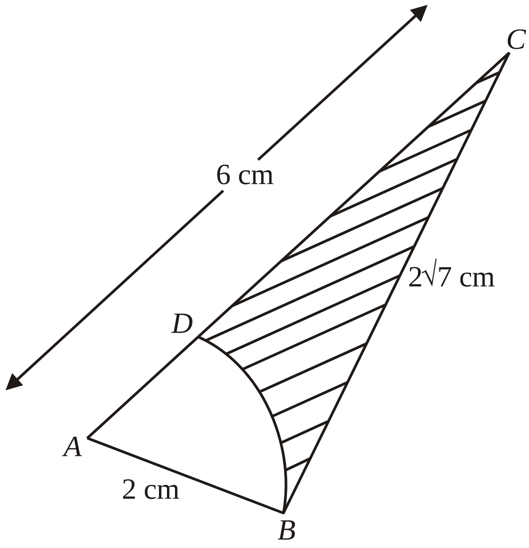
(b) the perimeter of *R*, giving your answer to 3 significant figures,

(4)

(c) the area of *R*, giving your answer to 3 significant figures.

(5)
(Total 11 marks)

4.



In ΔABC, AB = 2 cm, AC = 6 cm and $BC = 2\sqrt{7}$ cm.

(a) Use the cosine rule to show that $\angle BAC = \dfrac{\pi}{3}$ radians.

(3)

The circle with centre A and radius 2 cm intersects AC at the point D, as shown in the diagram above.

Calculate

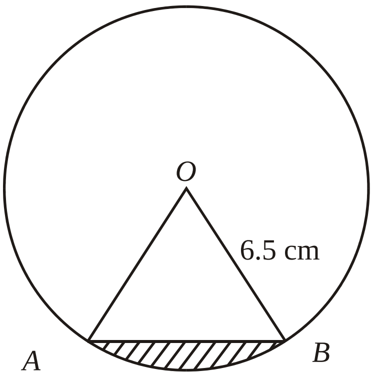
(b) the length, in cm, of the arc BD,

(2)

(c) the area, in cm^2, of the shaded region BCD.

(4)
(Total 9 marks)

5.



The diagram above shows the sector *AOB* of a circle, with centre *O* and radius 6.5 cm, and $\angle AOB$ = 0.8 radians.

(a) Calculate, in cm^2, the area of the sector *AOB*.

(2)

(b) Show that the length of the chord *AB* is 5.06 cm, to 3 significant figures.

(3)

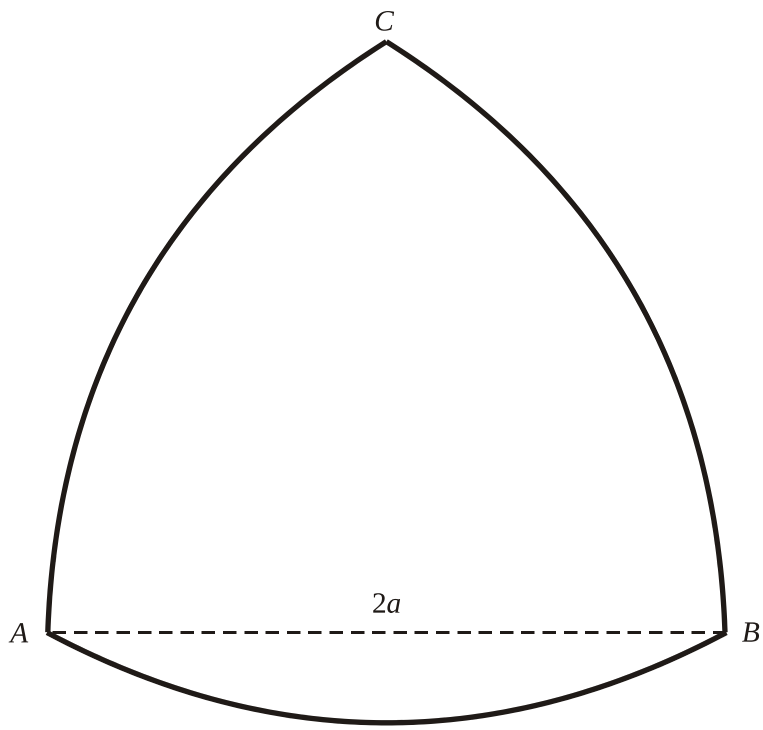
The segment *R*, shaded in the diagram above, is enclosed by the arc *AB* and the straight line *AB*.

(c) Calculate, in cm, the perimeter of *R*.

(2)

(Total 7 marks)

6.



A flat plate S, which is part of a child's toy, is shown in the diagram above. The points A, B and C are the vertices of an equilateral triangle and the distance between A and B is $2a$. The circular arc AB has centre C and radius $2a$. The circular arcs BC and CA have centres at A and B respectively and radii $2a$.

(a) Find, in terms of π and a, the perimeter of S.

(2)

(b) Prove that the area of the plate S is

$$2a^2(\pi - \sqrt{3}).$$

(6)
(Total 8 marks)

SOLUTIONS!

1. (a) $\frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times 1.5 = 15$ M1 A1

$r^2 = 20 = \sqrt{(4 \times 5)}$ $r = 2\sqrt{5}$ (*) A1 3

(b) $r\theta + 2r = 3\sqrt{5} + 4\sqrt{5} = 7\sqrt{5}$ cm (or 15.7, or a.w.r.t 15.65….) M1 A1 2

(c) ΔOAB: $\frac{1}{2}r^2 \sin\theta = 10 \sin 1.5$ (= 9.9749…) M1

Segment area = 15 – ΔOAB = 5.025 cm^2 M1 A1 3

[8]

2. (a) $r\theta = 45\theta = 63$, $\theta = 1.4$ (*) M1A1 2

M1 is for applying correct formula or quoting and attempting to use correct formula

(b) Area of sector $OAB = \frac{1}{2}r^2\theta = \frac{1}{2}45^2 \times 1.4$ (= 1417.5) M1A1

Area of triangle $OCD = \frac{1}{2}30^2 \times \sin 1.4$ (= 443.45) M1A1

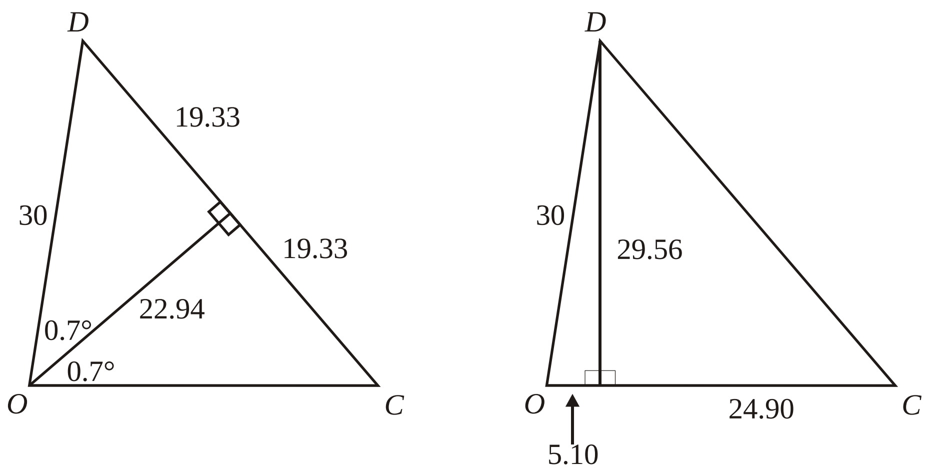
Shaded area = 1417.5 – 443.45…= 974 m^2 cao A1 5

For each area
M1 is for attempting to use correct formula or complete method in case of Δ (*)
A1 is for a numerically correct statement (answer is not required – just there as check)

Final A1 is for 974 only.

e.g. splitting triangle into two triangles:

For guidance



[7]

3.	(a)	$r\theta = 8 \times 0.7, = 5.6(cm)$	M1, A1	2
	(b)	$BC^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \cos 0.7$	M1	
		$\Rightarrow BC = 7.098$	A1	
		$\Rightarrow$ Perimeter = (a) + (11 – 8) + BC, = 15.7(cm)	M1, A1cao	4
	(c)	$\Delta = \frac{1}{2}ab\sin c = \frac{1}{2} \times 11 \times 8 \times \sin 0.7$, = AWRT 28.3	M1, A1	
		Sector = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times 0.7$	M1, A1	
		Area of R = 28.345….. – 22.4 = 5.9455 = 5.95(cm^2)	A1	5
				[11]
4.	(a)	$\cos A = \frac{6^2 + 2^2 - (2\sqrt{7})^2}{2 \times 6 \times 2}$	M1 A1	
		$\cos A = \frac{1}{2}$ $A = \frac{\pi}{3}$ radians (*)	A1	3
	(b)	$r\theta = \frac{2\pi}{3}$ (= 2.09) (Exact or at least 3 s.f.)	M1 A1	2
	(c)	Sector ABD: $\frac{1}{2}r^2\theta = \frac{1}{2} \times 2^2 \times \frac{\pi}{3}$ $\left(= \frac{2\pi}{3} \approx 2.094...\right)$	M1	
		Triangle ACB: $\frac{1}{2} \times 2 \times 6 \times \sin\frac{\pi}{3}$ $(= 3\sqrt{3} \approx 5.196\ldots)$	M1	
		Triangle – Sector = $3\sqrt{3} - \frac{2\pi}{3}$ (= 3.10175…)	M1 A1	4
		Allow 3.1 or a.w.r.t. 3.10		
				[9]
5.	(a)	$\frac{1}{2}r^2\theta = \frac{1}{2} \times 6.5^2 \times 0.8 = 16.9$ (a.w.r.t. if changed to degrees)	M1 A1	2
	(b)	$\sin 0.4 = \frac{x}{6.5}$, $x = 6.5 \sin 0.4$, (where x is half of AB)	M1, A1	
		(n.b. 0.8 rad = 45.8°)		
		$AB = 2x = 5.06$ (a.w.r.t.) (*)	A1	3
		<u>Alternative:</u> $AB^2 = 6.5^2 + 6.5^2 - 2 \times 6.5 \times 6.5 \cos 0.8$	[M1]	
		$AB = \sqrt{6.5^2 + 6.5^2 - 2 \times 6.5 \times 6.5 \cos 0.8}$	[A1]	
		$AB = 5.06$	[A1]	
	(c)	$r\theta + 5.06 = (6.5 \times 0.8) + 5.06 = 10.26$ (a.w.r.t) (or 10.3)	M1 A1	2
				[7]

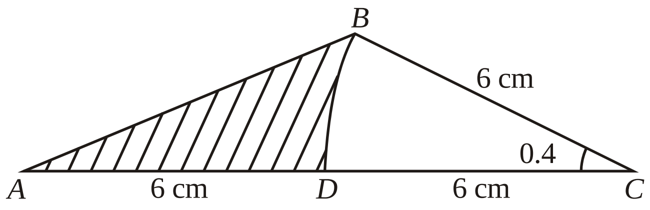
6.	(a)	Arc $AB = 2a \times \frac{\pi}{3}$ (using $r\theta$)	M1	
		Perimeter of $S = 3 \times \frac{2a\pi}{3} = 2a\pi$	A1	2
	(b)	Area of sector $ABC = \frac{1}{2}(2a)^2\frac{\pi}{3} = 2a^2 \frac{\pi}{3}$	B1	
		Area of triangle $ABC = \frac{1}{2}(2a)^2 \sin\frac{\pi}{3} = a^2\sqrt{3}$	M1 A1	
		Area of segment $= 2a^2 \frac{\pi}{3} - a^2\sqrt{3}$	M1	
		Area of S = 3 (Area of segment ABC) + (Area of triangle ABC)	M1	
		$= 2\pi a^2 - 3a^2\sqrt{3} + a^2\sqrt{3}$		
		$= 2a^2(\pi - \sqrt{3})$	A1	6
				[8]

Name……………………………………

HOMEWORK

DUE IN ON ……………………………………

7.



The diagram above shows a logo *ABD*.

The logo is formed from triangle *ABC*. The mid-point of *AC* is *D* and *BC* = *AD* = *DC* = 6 cm.
∠*BCA* = 0.4 radians. The curve *BD* is an arc of a circle with centre *C* and radius 6 cm.

(a) Write down the length of the arc *BD*.

(1)

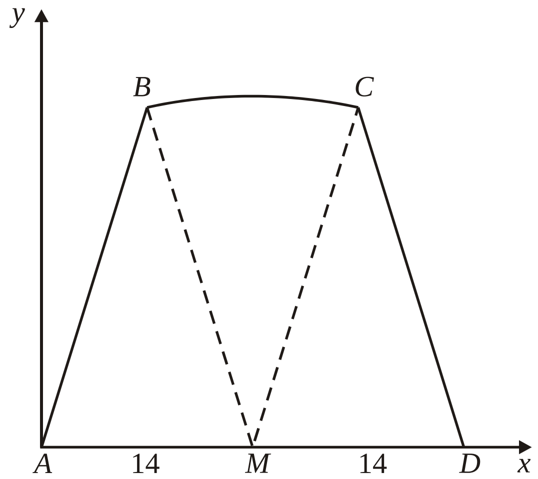
(b) Find the length of *AB*.

(3)

(c) Write down the perimeter of the logo *ABD*, giving your answer to 3 significant figures.

(1)
(Total 5 marks)

8.



The diagram above shows the cross-section *ABCD* of a chocolate bar, where *AB*, *CD* and *AD* are straight lines and *M* is the mid-point of *AD*. The length *AD* is 28 mm, and *BC* is an arc of a circle with centre *M*.

Taking *A* as the origin, *B*, *C* and *D* have coordinates (7, 24), (21, 24) and (28, 0) respectively.

(a) Show that the length of *BM* is 25 mm.

(1)

(b) Show that, to 3 significant figures, $\angle BMC = 0.568$ radians.

(3)

(c) Hence calculate, in mm^2, the area of the cross-section of the chocolate bar.

(5)

Given that this chocolate bar has length 85 mm,

(d) calculate, to the nearest cm^3, the volume of the bar.

(2)
(Total 11 marks)