REVISION QUESTIONS

SEQUENCES, SERIES, BINOMIAL AND BASIC DIFFERENTIATION

ARITHMETIC SEQUENCES
BASIC QUESTIONS

1) An arithmetic sequence is defined a=5 and d=3. Write down the first 6 terms.

2) An arithmetic sequence is defined a=13 and d=-10. Write down the first 6 terms.

3) An arithmetic sequence is defined a=1000 and d=55. Write down the first 6 terms.

4) Write down the value of a and d for the following sequences:

   a) 4, 6, 8, 10, …….

   b) 6, 16, 26, 36, …..

   c) 50, 49.5, 49, 48.5, …….

   d) $\frac{2}{5}$, $\frac{4}{5}$, $1\frac{1}{5}$, …..

5) The sequence 90, 88.5, 87, 85.5,……..is arithmetic.

   a) Find the 20^{th} term.

   b) Find an expression for the n*th* term.

   c) Find the value of n for which the n*th* term in the sequence is 42.

ARITHMETIC SERIES
BASIC QUESTIONS

1. The first term of an arithmetic progression is 6 and the fifth term is 12. The progression has n terms and the sum of all the terms is 90. Find the value of n.

[4]

2. The 1st term of an arithmetic progression is a and the common difference is d, where $d \neq 0$.

(a) Write down expressions, in terms of a and d, for the 5th term and the 15th term.

[1]

(b) The first two terms in an arithmetic progression are 5 and 9. The last term in the progression is the only term which is greater than 200. Find the sum of all the terms in the progression.

[4]

1. 1^{st} term $= a = 6$
5^{th} term $= a + 4d = 12$
$\rightarrow d = 1.5$
$S_n = \frac{n}{2}(12 + (n-1)1.5) = 90$
$\rightarrow n^2 + 7n - 120 = 0$
$\rightarrow n = 8$

2. (a) $a + 4d$ and $a + 14d$

(b) $a = 5$, $d = 4$
$200 = a + (n-1)d$
or T.I.
50 terms in the progression
$\rightarrow 5150$

GEOMETRIC SERIES
BASIC QUESTIONS

$$S_n = \frac{a(1-r^n)}{1-r} \qquad S_\infty = \frac{a}{1-r}$$

Find the total of these geometric series:

1. $1 + 2 + 4 + 8 + 16 + 32 + 64 + 128$

2. $32 + 16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

3. $4 - 12 + 36 - 108 + 324 - 972$

4. $3125 + 625 + 125 + 25 + 5 + \cdots \qquad n = \infty$

5. $192 + 48 + 12 + 3 + \cdots \qquad n = \infty$

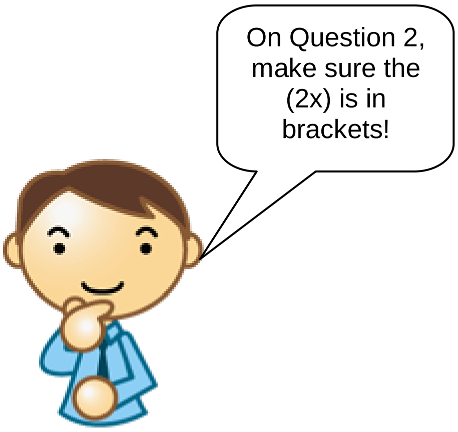
BINOMIAL
BASIC QUESTIONS

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{1}a^{n-1}b + \ldots\ldots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

Where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{(n-r)!r!}$ which is the n^{th} row and r^{th} position of Pascal's Triangle

1.
   a. Use the binomial theorem to work out $(a+b)^3$

   b. Now use your answer to work out $(x+2)^3$



2.
   a. Use the binomial theorem to work out $(a+b)^4$

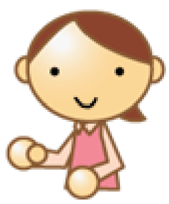
   b. Now use your answer to work out $(2x+3)^4$

3.
   a. Use the binomial theorem to work out $(a+b)^5$

   b. Now use your answer to work out $\left(\dfrac{x}{2}+1\right)^5$

4.
   a. Use the binomial theorem to work out $(a+b)^5$

   b. Now use your answer to work out $(x-2)^5$



FUNCTIONS
BASIC QUESTIONS

1. Give an example of a one-to-one function and a many-to-one function

2. If $f(x) = 3x + 2, (Domain: x > 0)$ and $g(x) = x^2 + 5, (Domain: x \in \mathbb{R})$
   a. What sort of functions are these?

   b. What is the range of $f(x)$?

   c. What is the range of $g(x)$?

   d. What is the inverse of $f(x)$?

   e. Why doesn't $g(x)$ have an inverse?

   f. What is the value of $fg(5)$?

DIFFERENTIATION BASIC QUESTIONS

1) Find $\dfrac{dy}{dx}$ for the following curves.

(a) $y = 2x^2$ **(b)** $y = 4x^3$ **(c)** $y = 3x^5$ **(d)** $y = 12x^{10}$ **(e)** $y = -4x^2$

(f) $y = -3x^{-1}$ **(g)** $y = 3x$ **(h)** $y = 4x$ **(i)** $y = -5x$ **(j)** $y = -4x^2$

(k) $y = -3x^{-2}$ **(l)** $y = 12x - 1$ **(m)** $y = 3x + 2$ **(n)** $y = x^2 + x$ **(o)** $y = 2x^2 + 4x^3$

(p) $y = 2x^2 + 3x + 4$ **(q)** $y = 5x^2 - 6x - 2$ **(r)** $y = 1 - x^2$

(s) $y = 2x - x^3 + 4$ **(t)** $y = (x-1)(x+1)$ **(u)** $y = x(x+3)(x-1)$

(v) $y = \dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3}$ **(w)** $y = x - 3x^{-2}$ **(x)** $y = \dfrac{3}{x^2} - \dfrac{2}{x} + 3$

(y) $y = 4x^2 + (2x-1)^2$ **(z)** $y = \left(x - \dfrac{1}{x}\right)^2$.

2) Find the stationary points of $y = x^3 - 3x^2 - 45x + 7$ and describe their nature.

MIXED EXAM STYLE QUESTIONS

1. A geometric progression has 6 terms. The first term is 192 and the common ratio is 1.5. An arithmetic progression has 21 terms and common difference 1.5. Given that the sum of all the terms in the geometric progression is equal to the sum of all the terms in the arithmetic progression, find the first term and the last term of the arithmetic progression.

[6]

2. (a) Find the sum of all the integers between 100 and 400 that are divisible by 7.

[4]

(b) The first three terms in a geometric progression are 144, x and 64 respectively, where x is positive. Find

(i) the value of x,

(ii) the sum to infinity of the progression.

[5]

3. The first term of a geometric progression is 81 and the fourth term is 24. Find

(i) the common ratio of the progression, **[2]**

(ii) the sum to infinity of the progression. **[2]**

The second and third terms of this geometric progression are the first and fourth terms respectively of an arithmetic progression.

(iii) Find the sum of the first ten terms of the arithmetic progression. **[3]**

4. The 1st term of an arithmetic progression is a and the common difference is d, where $d \neq 0$.

(i) Write down expressions, in terms of a and d, for the 5th term and the 15th term. **[1]**

The 1st term, the 5th term and the 15th term of the arithmetic progression are the first three terms of a geometric progression.

(ii) Show that $3a = 8d$. **[3]**

(iii) Find the common ratio of the geometric progression. **[2]**

5. Each year a company gives a grant to a charity. The amount given each year increases by 5% of its value in the preceding year. The grant in 2001 was \$5000. Find

(i) the grant given in 2011,

[3]

(ii) the total amount of money given to the charity during the years 2001 to 2011 inclusive.

[2]

6. (a) Find the sum to infinity of the geometric progression with first three terms 0.5, 0.5^3 and 0.5^5.

[3]

(b) The first two terms in an arithmetic progression are 5 and 9. The last term in the progression is the only term which is greater than 200. Find the sum of all the terms in the progression.

[4]

7. The first term of an arithmetic progression is 6 and the fifth term is 12. The progression has n terms and the sum of all the terms is 90. Find the value of n.

[4]

8. (i) Find the first 3 terms in the expansion of $(2 - x)^6$ in ascending powers of x.

[3]

(ii) Find the value of k for which there is no term in x^2 in the expansion of $(1 + kx)(2 - x)^6$.

[2]

9. The first three terms in the expansion of $(2 + ax)^n$, in ascending powers of x, are $32 - 40x + bx^2$. Find the values of the constants n, a and b.

[5]

10. (i) Find the first 3 terms in the expansion, in ascending powers of x, of $(2 + x^2)^5$.

[3]

(ii) Hence find the coefficient of x^4 in the expansion of $(1 + x^2)^2(2 + x^2)^5$.

[3]

11. Find the value of the coefficient of x^2 in the expansion of $\left(\frac{x}{2}+\frac{2}{x}\right)^6$.

[3]

12. The function f is defined by $f : x \mapsto 2x^2 - 12x + 13$ for $0 \leq x \leq A$, where A is a constant.

(i) Express f(x) in the form $a(x + b)^2 + c$, where a, b and c are constants.

[3]

(ii) State the value of A for which the graph of $y = f(x)$ has a line of symmetry.

[1]

(iii) When A has this value, find the range of f.

[2]

The function g is defined by $g : x \mapsto 2x^2 - 12x + 13$ for $x \geq 4$.

(iv) Explain why g has an inverse.

[1]

(v) Obtain an expression, in terms of x, for $g^{-1}(x)$.

[3]

13. The function f is defined by

$$f : x \mapsto 3x - 2 \text{ for } x \in \mathbb{R}.$$

(i) Sketch, in a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the two graphs.

[2]

The function g is defined by

$$g : x \mapsto 6x - x^2 \text{ for } x \in \mathbb{R}.$$

(ii) Express $gf(x)$ in terms of x, and hence show that the maximum value of $gf(x)$ is 9.

[5]

The function h is defined by

$$h : x \mapsto 6x - x^2 \text{ for } x \geq 3.$$

(iii) Express $6x - x^2$ in the form $a - (x - b)^2$, where a and b are positive constants.

[2]

(iv) Express $h^{-1}(x)$ in terms of x.

[3]

14. Functions f and g are defined by

$$f : x \mapsto 4x - 2k \text{ for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$

$$g : x \mapsto \frac{9}{2-x} \text{ for } x \in \mathbb{R}, x \neq 2.$$

(i) Find the values of **k** for which the equation fg(x) = x has two equal roots.

[4]

(ii) Determine the roots of the equation fg(x) = x for the values of **k** found in part (i).

[3]

15. Functions f and g are defined by

$$f : x \mapsto k - x \quad \text{for } x \in \mathbb{R}, \text{ where k is a constant,}$$

$$g : x \mapsto \frac{9}{x+2} \text{ for } x \in \mathbb{R}, x \neq -2.$$

(i) Find the values of k for which the equation f(x) = g(x) has two equal roots and solve the equation f(x) = g(x) in these cases.

[6]

(ii) Solve the equation fg(x) = 5 when k = 6.

[3]

(iii) Express $g^{-1}(x)$ in terms of x.

[2]

16. The function f is defined by $f : x \mapsto x^2 - 3x$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $f(x) > 4$.

[3]

(ii) Express $f(x)$ in the form $(x - a)^2 - b$, stating the values of a and b.

[2]

(iii) Write down the range of f.

[1]

(iv) State, with a reason, whether f has an inverse.

[1]

The function g is defined by $g : x \mapsto x - 3\sqrt{x}$ for $x \geq 0$.

(v) Solve the equation $g(x) = 10$.

[3]

17. A curve has equation $y = \frac{4}{\sqrt{x}}$.

(i) The normal to the curve at the point (4, 2) meets the x-axis at P and the y-axis at Q. Find the length of PQ, correct to 3 significant figures.

[6]

18. A curve has equation $y = \frac{k}{x}$, Given that the gradient of the curve is –3 when $x = 2$, find the value of the constant k.

[3]

MIXED EXAM STYLE QUESTION SOLUTIONS

1. GP $a = 192$, $r = 1.5$, $n = 6$
AP $a = a$, $d = 1.5$, $n = 21$

S_6 for GP = $192(1.5^6 - 1) \div 0.5$
= 3990 M1
Correct sum formula used.

S_{21} for AP = $\frac{21}{2}$ (2a + 20 × 1.5) M1
Correct sum formula used.

DM1

Equate and solve → $a = 175$

M1 A1

Correct formula used.

21^{st} term in AP = a + $20d$ = 205
(or from 3990 = 21(a + l)/2

[6]

2. (a) $a = 105$ B1
co

Either $l = 399$ or $d = 7$ B1
co

$n = 43$ B1
co

→ 10836 B14
co

(b) $r^2 = 64/144 \to r = \frac{2}{3}$ M1
award in either part

(i) Either $x = ar \to x = 96$
or $\frac{144}{x} = \frac{x}{64} \to x = 96$ M1A1
either method ok

(ii) Use of $S_\infty = \frac{a}{1-r}$ M1
Used with his a and r

→ 432 A15
Co
(nb do not penalise if r and l or x negative as well as positive.)

3. (i) $a = 81$, $ar^3 = 24$ M1
Valid method for r.

$\to r^3 = 24/81 \to r = \frac{2}{3}$ or 0.667 A12

(ii) $S_\infty = \dfrac{a}{1-r} = 81 \div \frac{1}{3} = 243$ — M1 A1√2

Correct formula. √ for his *a* and *r*, providing $-1 < r < 1$.

(iii) 2nd term of GP = $ar = 81 \times \frac{2}{3} = 54$

3rd term of GP = $ar^2 = 36$ — M1

Finding the 2nd and 3rd terms of GP.

$\rightarrow 3d = -18$ $(d = -6)$ — M1

M for finding *d* + correct S_{10} formula. Co

$\rightarrow S_{10} = 5 \times (108 - 54) = 270$ — A13

[7]

4. (i) $a + 4d$ and $a + 14d$ — B11

Both correct.

(ii) $a + 4d = ar$, $a + 14d = ar^2$ — M1

Correct first step – awards the mark for both of these starts.

(iii) $r = \dfrac{a+4d}{a}$ or $\dfrac{a+14d}{a+4d} = 2.5$ — M1A12

Statement + some substitution. Co

[6]

5. (i) r = 1.05 with GP

2011 is 11 years. — B1

Anywhere in the question. This could be marked as 2 + 3

Uses ar^{n-1} — M1

Allow if correct formula with $n = 10$

→ \$8 144 (or 8140) — A13

co. (allow 3sf)

(ii) Use of S_n formula — M1

Allow if used correctly with 10 or 11.

→ \$71 034 — A12

co (or 71 000)

[5]

6. (a) $a = 0.5$, $r = 0.5^2$ — B1

For both *a* and *r*.

Uses correct formula = $0.5 \div 0.75$ — M1

Uses correct formula with some *a*, *r*.

$\rightarrow S_\infty = \frac{2}{3}$ (or 0.667) — A13

(b) $a = 5$, $d = 4$

Uses $200 = a + (n-1)d$ or T.I. — M1

Attempt at finding the number of terms.

50 terms in the progression — A1

co.

Use of correct Sum formula Correct formula (could use the last term (201)).	M1	
$\rightarrow 5150$	A14	
		[7]

7. 1st term = $a = 6$
5th term = $a + 4d = 12$
$\rightarrow d = 1.5$ — B1
Correct value of d

$S_n = \frac{n}{2}(12 + (n-1)\,1.5) = 90$ — M1
Use of correct formula with his d

$\rightarrow n^2 + 7n - 120 = 0$ — DM1
Correct method for soln of quadratic

$\rightarrow n = 8$ — A1
Co (ignore inclusion of $n = -15$)

[4]

8. (i) $(2-x)^6 = 64 - 192x + 240x^2$ — $3 \times$ B13
One for each term. Allow 2^6.

(ii) $(1 + kx)(2 - x)^6$
coeff of $x^2 = 240 - 192k$ — M1
Must be considering sum of 2 terms.
ft for this expansion.

$= 0 \rightarrow k = 5/4$ or 1.25 — A1√2
(allow M1 if looking for coeff of x).

[5]

9. $(2 + ax)^n$
1st term = $2^n = 32 \rightarrow n = 5$ — B1
co

2nd term = $n.2^{n-1}(ax) = -40x$ — M1
Allow for both binomial coefficients

3rd term = $n(n-1).\frac{1}{2}.2^{n-2}.(ax)^2$ — M1
Allow for one power of 2 and ax

$\rightarrow a = -\frac{1}{2}$ — A1
co

$\rightarrow b = 20$ — A1
co

[5]

10. (i) $(2 + x^2)^5 = 2^5 + 5.2^4.x^2 + 10.2^3.x^4$
$\rightarrow 32 + 80x^2 + 80x^4$
If coeffs ok but x and x^2, allow B1 special case. Allow 80, 80 if in (ii). — $3 \times$ B13

(allow 2^5 for 32)

(ii) $(1 + x^2)^2 = 1 + 2x^2 + x^4$ — B1
Anywhere.

Product has 3 terms in x^4 M1
Must be attempt at more than 1 term.

$\rightarrow 80 + 160 + 32 = 272$ A1√3
For follow-through on both expansions,
providing there are 3 terms added.

[6]

11. $\left(\frac{x}{2}+\frac{2}{x}\right)^6$

Term in x^2 $\left(\frac{x}{2}\right)^4\left(\frac{2}{x}\right)^2 \times 15$ M1
Correct term – needs powers 4 and 2

A1
For × 15

Coeff = $\frac{15}{4}$ or 3.75 A1
Ignore inclusion of x^2

[3]

12. (i) $2x^2 - 12x + 13 = 2(x-3)^2 - 5$ 3 × B13
Allow even if *a*, *b*, *c* not specifically quoted.

(ii) Symmetrical about $x = 3$. $A = 6$. B1√1
For 2 × his $(-b)$.

(iii) One limit is –5 B1√
For his *c*.

Other limit is 13 B12
co.

(iv) Inverse since 1:1 (4 > 3). B11
Valid argument.

(v) Makes *x* the subject of the equation M1
Attempts to change the formula.

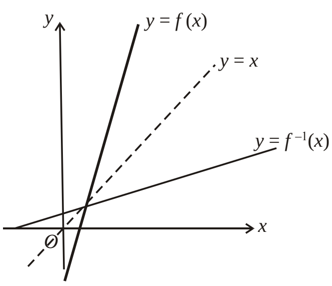
Order of operations correct DM1
"+5", ÷2, √, +3. Allow for simple algebraic
slips such as – 5 for +5 etc.

$\rightarrow \sqrt{\frac{x+5}{2}} + 3$ A13
co – as a function of *x*, not *y*.
condone ±.

[10]

13. $f : x \rightarrow 3x - 2$

(i)



	Graph of $y = 3x - 2$	B1	
	Evidence of mirror image in $y = x$ or graph of 1/3 $(x + 2)$. Whichever way, there must be symmetry shown or quoted or implied by same intercepts.	B12	
(ii)	$gf(x) = 6(3x - 2) - (3x - 2)^2$ Must be gf, not fg	M1	
	$= -9x^2 + 30x - 16$ Co	A1	
	$d/dx = -18x + 30$ Differentiates or completes square	M1	
	$= 0$ when $x = 5/3$ Sets to 0, solves and attempts to find y	DM1	
	$\rightarrow$ Max of 9 All ok – answer was given	A15	
	$(gf(x) = 9 - (3x - 5)^2 \rightarrow$ Max 9)		
(iii)	$6x - x^2 = 9 - (x - 3)^2$ Does not need a or b.	B1, B12	
(iv)	$y = 9 - (3 - x)^2$ Order of operations in making x subject	M1	
	$3 - x = \pm\sqrt{9 - y}$ Interchanging x and y	DM1	
	$\rightarrow h^{-1}(x) = 3 + \sqrt{(9 - x)}$ Allow if ± given	A13	
	(Special case $\rightarrow$ if correct with y instead of x, give 2 out of 3)		
			[12]

14. $f : x \rightarrow 4x - 2k$, $g : x \mapsto \dfrac{9}{2-x}$

(i)	$fg(x) = \dfrac{36}{2-x} - 2k = x$ Knowing to put g into f (not gf)	M1	
	$x^2 + 2kx - 2x + 36 - 4k$ Correct quadratic.	A1	
	$(2k - 2)^2 = 4(36 - 4k)$ Any use of $b^2 - 4ac$ on quadratic = 0	M1	
	$k = 5$ or -7 Both correct.	A14	
(ii)	$x^2 + 8x + 16 = 0$, $x^2 - 16x + 64 = 0$	M1	

Substituting one of the values of k.

$x = -4$ or $x = 8$. A1 A13

15. **f :** $x \mapsto k - x$

g : $x \mapsto \dfrac{9}{x+2}$

(i) $k - x = \mapsto \dfrac{9}{x+2}$

$\rightarrow x^2 + (2-k)x + 9 - 2k = 0$ M1
Forming a quadratic equation

Use of $b^2 - 4ac$ M1
Use of $b^2 - 4ac$ on quadratic = 0

$\rightarrow a = 4$ or -8 DM1 A1
DM1 for solution. A1 both correct.

$k = 4$, roots is $\dfrac{-b}{2a} = 1$ M1
Any valid method.

$k = -8$, root is -5. A16
Both correct.

(ii) $\text{fg}(x) = 6 - \dfrac{9}{x+2}$ M1
Must be fg, not for gf.

Equates and solves with 5 DM1
Reasonable algebra.

$x = 7$ A13
co.

[or $\text{fg}(x) = 5 \rightarrow \text{g}(x) = 1 \rightarrow x = 7$]
[$\text{g}(x) = 1$ M1 $\rightarrow x$ DM1 $x = 7$ A1]

(iii) $y = \dfrac{9}{x+2} \rightarrow x = \dfrac{9}{y} - 2$ M1
Virtually correct algebra, Allow + for –.

$\text{g}^{-1}(x) = \dfrac{9}{x} - 2$ or $\dfrac{9-2x}{x}$ A12
Correct and in terms of x.

[11]

16. (i) $x^2 - 3x - 4 \rightarrow -1$ and 4 M1A1
Solving Quadratic = 0. Correct values.

$\rightarrow x < -1$ and $x > 4$ A13
co. – allow ≤ and /or ≥, $4 < x < -1$ ok

(ii) $x^2 - 3x = (x - \dfrac{3}{2})^2 + -\dfrac{9}{4}$ B1 B12

B1 for $\dfrac{3}{2}$. B1 for $\dfrac{9}{4}$.

(iii) $\text{f}(x)$ (or y) $\geq -\dfrac{9}{4}$ B1√1
√ for $\text{f}(x) \geq$ "$-b$".

(iv) No inverse – not 1 : 1. B1 1
Independent of previous working.

(v) Quadratic in $\sqrt{x}$ M1
Recognition of "Quadratic in $\sqrt{x}$"

Solutions → $\sqrt{x} = 5$ or –2 DM1
Method of solution.

→ $x = 25$ A1 3
co. Loses this mark if other answers given. Nb ans only full marks.

[10]

17. $y = \dfrac{4}{\sqrt{x}}$

(i) $dy/dx = -2x^{-1.5}$ M1
Reasonable attempt at differentiation with his power of x.

$= -\frac{1}{4}$ A1
CAO

m of normal = 4 M1
Use of $m_1 m_2 = -1$ even if algebraic.

Eqn of normal $y - 2 = 4(x - 4)$
P (3.5, 0) and Q (0, −14) M1
Use of equation for a straight line + use of $x = 0$ and $y = 0$.

Length of $PQ = \sqrt{(3.5^2 + 14^2)}$ M1
Needs correct formula or method.
$= 14.4$ A1 6
CAO

(ii) Area = $\int_1^4 4x^{-0.5}\,dx = [\dfrac{4x^{0.5}}{0.5}]$ M1 A1
Attempt at integration. Correct unsimplified.

$= [8\sqrt{x}] = 16 - 8 = 8$ DM1A1 4
Correct use of limits. CAO

[10]

18. $\dfrac{dy}{dx} = -kx^{-2}$ B1
Negative power ok.

Puts $x = 2$, $m = -3$ M1
Subs $x = 2$ into his dy/dx.

→ $k = 12$ A1
co.

[3]