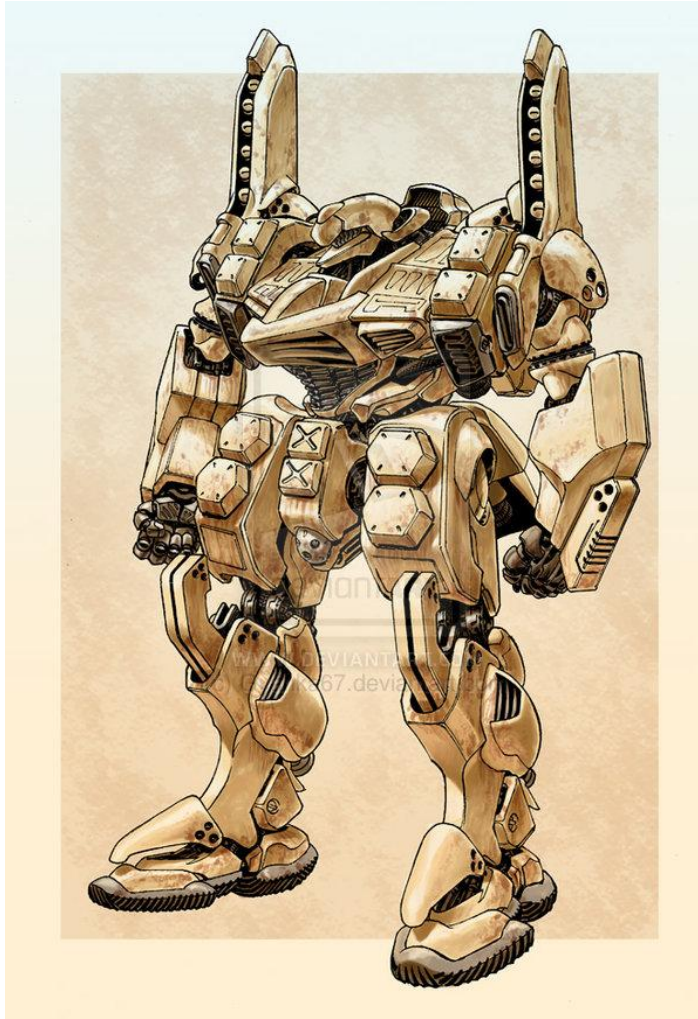# P1

TRIG PAST PAPER QUESTIONS

1. (i) Show that the equation $\sin\theta + \cos\theta = 2(\sin\theta - \cos\theta)$ can be expressed as $\tan\theta = 3$. **[2]**

(ii) Hence solve the equation $\sin\theta + \cos\theta = 2(\sin\theta - \cos\theta)$, for $0° \leq \theta \leq 360°$.

[2]

2. A function f is defined by $f : x \mapsto 3 - 2\sin x$, for $0° \leq x \leq 360°$.

(i) Find the range of f.

[2]

(ii) Sketch the graph of $y = f(x)$.

[2]

A function g is defined by $g : x \mapsto 3 - 2\sin x$, for $0° \leq x \leq A°$, where A is a constant.

(iii) State the largest value of A for which g has an inverse.

[1]

(iv) When A has this value, obtain an expression, in terms of x, for $g^{-1}(x)$.

[2]

3. Solve the equation $3 \sin^2\theta - 2\cos\theta - 3 = 0$, for $0° \leq \theta \leq 180°$.

[4]

4. Solve the equation

$$\sin 2x + 3\cos 2x = 0,$$

for $0° \leq x \leq 180°$.

[4]

5. Given that $x = \sin^{-1}\left(\dfrac{2}{5}\right)$ find the exact value of

(i) $\cos^2 x$,

[2]

(ii) $\tan^2 x$.

[2]

6. Prove the identity $\dfrac{1-\tan^2 x}{1+\tan^2 x} \equiv 1 - 2\sin^2 x.$

[4]

7. How much wood would a wood chuck chuck if a wood chuck could chuck wood?

8. (i) Show that the equation $3 \sin x \tan x = 8$ can be written as $3 \cos^2 x + 8 \cos x - 3 = 0$.

[3]

(ii) Hence solve the equation $3 \sin x \tan x = 8$ for $0° \leq x \leq 360°$.

[3]

9. Prove the identity $\dfrac{\sin x}{1 - \sin x} - \dfrac{\sin x}{1 + \sin x} \equiv 2 \tan^2 x$.

[3]

1. (i) $s + c = 2s - 2c \rightarrow s = 3c$ M1

                       Use of $t = s/c$ + collection $\rightarrow \tan \theta = k$.

         $\rightarrow \tan \theta = 3$ A12

         Algebra needed to reduce to this form.

      (ii) $\rightarrow \theta = 71.6°$ or $251.6°$ B1 B1√2

         B1√ for $180 + ...$as only soln in range.

                                                            [4]

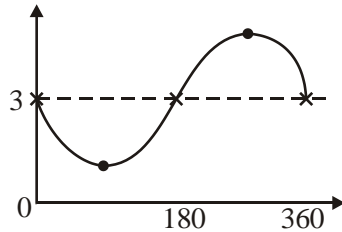**2.**   f : $x \rightarrow 3 - 2\sin x$ for $0° \leq x \leq 360°$.

(i)   Range $1 \leq f(x) \leq 5$ B2,1,0   2
Needs 1, 5, $\leq$. One off for each error.

(ii)

g: $x \rightarrow 3 - 2\sin x$ for $0^o \leq x \leq A^o$ B2,1,0   2

Must be exactly 1 full oscillation – this
overrides the rest.
Starts and ends at 3.
Correct shape needed.
Curves, not blatant lines.

(iii)   Maximum value of $A = 90$ or $\frac{1}{2}\pi$ B1   1
CAO

(iv) $y = 3 - 2\sin x$ M1
Attempt to make x the subject and then
to replace x by y. Needs $\sin^{-1}$ ().

$g^{-1}(x) = \sin^{-1}\left(\dfrac{3-x}{2}\right)$ A1   2

Everything correct inc $\sin^{-1}$.
Allow these marks anywhere.

[7]

7

3. $3\sin^2\theta - 2\cos\theta - 3 = 0$

Use of $s^2 + c^2 = 1$ M1
Use of $s^2 + c^2 = 1$ to eliminate sine.

$3\cos^2\theta + 2\cos\theta = 0$ A1
Correct equation

$\cos\theta = 0$, $\theta = 90°$ B1
Co.

or $\cos\theta = -2/3$, $\theta = 131.8°$ A1
Co. (to 1 d.p or more – there must be
only this answer in the range 0 to 180))

                                                      [4]

4. $\tan 2x = -3$ M1
Use of $\tan = \sin/\cos$ with "$2x$"

$2x = 180 - 71.6$
or $2x = 360 - 71.6$ DM1
"$2x$" in second quadrant.

$\rightarrow x = 54.2°$ or $144.2°$ A1 A1√
co. For $90 + 1^{\text{st}}$ answer.

                                                        [4]

5. $x = \sin^{-1}\dfrac{2}{5} \rightarrow \sin x = \dfrac{2}{5}$

(i) $\cos^2 x = 1 - \sin^2 x = \dfrac{21}{25}$ M1 A12

         Formula only-use of $90°$ triangle ok
         co – loses if decimals blatantly used

(ii) $\tan^2 x = \dfrac{\sin^2 x}{\cos^2 x} = \dfrac{4}{21}$ M1 A12

         Formual only – or use of triangle ok
         Correct from his answer to(i).

                                                         [4]

6. Use of $t = s/c$ M1
tan completely removed

$\to (c^2 - s^2) \div (c^2 + s^2)$ A1
May omit the denominator (= 1)

Use of $c^2 + s^2 = 1$ M1
Whenever used appropriately

$\to (c^2 - s^2) \to 1 - 2\sin^2 x$ A1
ag Beware fortuitous answers

                 [4]

7. A wood chuck would chuck all the wood he could chuck if a wood chuck could chuck wood.

8. (i) $3\sin x \tan x = 8$
       Uses tan = sinθ + cosine   M1
       Replaces t by s/c

       Uses $\sin^2 = 1 - \cos^2$ M1
       Uses $\sin^2 = 1 - \cos^2$ for eqn in cosine.

       $\to 3\cos^2 x + 8\cos - 3 = 0$ A13
       Answer given.

   (ii) $(3c - 1)(c + 3) = 0$ or formula
       $\cos x = \frac{1}{3}$ as only solution.   M1
       Correct means of solution of quad.

       $x = 70.5°$ or $289.5°$ only.   A1 A1√3
       co. For $360° - 1^{st}$ ans + no other in
       range.

                 [6]

9. $\dfrac{s}{1-s} - \dfrac{s}{1+s} = \dfrac{2s^2}{1-s^2}$ B1
Correct algebra

Use of $1 - s^2 = c^2$ M1
Use of this formula.

$\to \dfrac{2s^2}{c^2}$
$\to 2t^2$ A1
Evidence of tan = sin/cos and everything
completed accurately.