PI VECTOR PAST PAPER QUESTIONS

1. Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

(i) Use a scalar product to find angle AOB, correct to the nearest degree.

[4]

(ii) Find the unit vector in the direction of $\overrightarrow{AB}$.

[3]

(iii) The point C is such that $\overrightarrow{OC} = 6\mathbf{j} + p\mathbf{k}$, where p is a constant. Given that the lengths of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are equal, find the possible values of p.

[4]

2. Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} - 8\mathbf{j} + 4\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 7\mathbf{i} + 2\mathbf{j} - \mathbf{k}.$$

(i) Find the value of $\overrightarrow{OA} . \overrightarrow{OB}$ and hence state whether angle AOB is acute, obtuse or a right angle.

[3]

(ii) The point X is such that $\overrightarrow{AX} = \frac{2}{5}\overrightarrow{AB}$. Find the unit vector in the direction of OX.

[4]

3. Relative to an origin O, the position vectors of points P and Q are given by

$$\overrightarrow{OP} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OQ} = \begin{pmatrix} 2 \\ 1 \\ q \end{pmatrix},$$

where q is a constant.

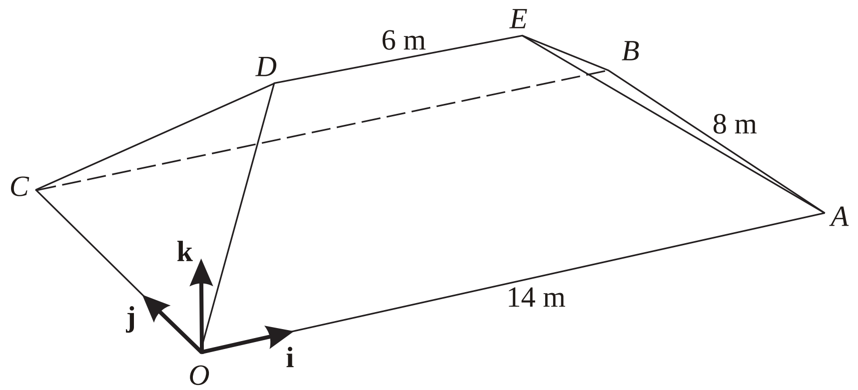
(i) In the case where $q = 3$, use a scalar product to show that $\cos POQ = \frac{1}{7}$.

[3]

(ii) Find the values of q for which the length of $\overrightarrow{PQ}$ is 6 units.

[4]

4.



The diagram shows the roof of a house. The base of the roof, *OABC*, is rectangular and horizontal with *OA* = *CB* = 14 m and *OC* = *AB* = 8 m. The top of the roof *DE* is 5 m above the base and *DE* = 6 m. The sloping edges *OD*, *CD*, *AE* and *BE* are all equal in length.

Unit vectors **i** and **j** are parallel to *OA* and *OC* respectively and the unit vector **k** is vertically upwards.

(i) Express the vector $\overrightarrow{OD}$ in terms of **i**, **j** and **k**, and find its magnitude. **[4]**

(ii) Use a scalar product to find angle *DOB*. **[4]**

5. The position vectors of points A and B are $\begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$ respectively, relative to an origin O.

(i) Calculate angle AOB.

[3]

(ii) The point C is such that $\overrightarrow{AC} = 3\overrightarrow{AB}$. Find the unit vector in the direction of $\overrightarrow{OC}$.

[4]

6. Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}.$$

(i) Given that C is the point such that $\overrightarrow{AC} = 2\overrightarrow{AB}$, find the unit vector in the direction of $\overrightarrow{OC}$.

[4]

The position vector of the point D is given by $\overrightarrow{OD} = \begin{pmatrix} 1 \\ 4 \\ k \end{pmatrix}$, where k is a constant, and it is given that $\overrightarrow{OD} = m\overrightarrow{OA} + n\overrightarrow{OB}$, where m and n are constants.

(ii) Find the values of m, n and k.

[4]

7. The diagram shows a cube *OABCDEFG* in which the length of each side is 4 units. The unit vectors **i, j** and **k** are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The mid-points of *OA* and *DG* are *P* and *Q* respectively and *R* is the centre of the square face *ABFE*.

(i) Express each of the vectors $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ in terms of **i, j** and **k**. **[3]**

(ii) Use a scalar product to find angle *QPR*. **[4]**

(iii) Find the perimeter of triangle *PQR*, giving your answer correct to 1 decimal place. **[3]**

8. Relative to an origin O, the position vectors of points A and B are $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} - 2\mathbf{j} + p\mathbf{k}$ respectively.

(i) Find the value of p for which OA and OB are perpendicular.

[2]

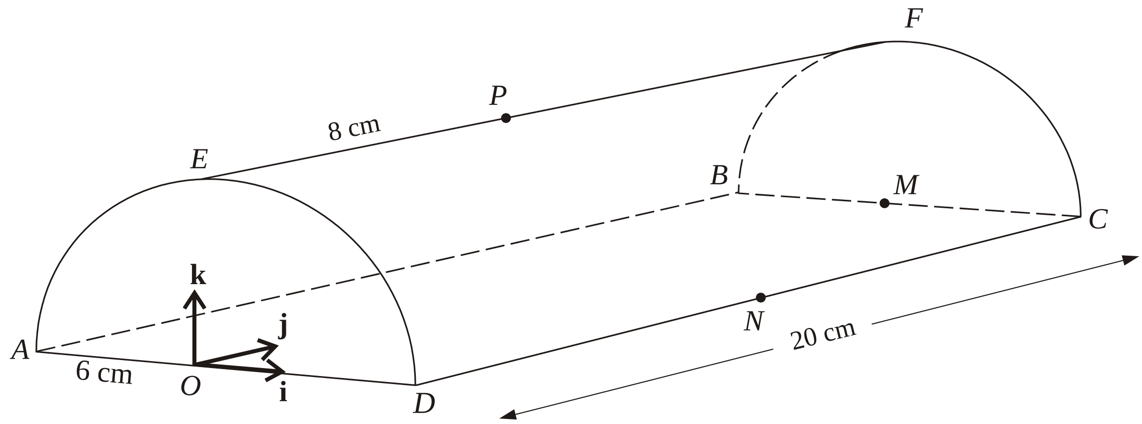
(ii) In the case where $p = 6$, use a scalar product to find angle AOB, correct to the nearest degree.

[3]

(iii) Express the vector $\overrightarrow{AB}$ is terms of p and hence find the values of p for which the length of AB is 3.5 units.

[4]

9.



The diagram shows a semicircular prism with a horizontal rectangular base *ABCD*. The vertical ends *AED* and *BFC* are semicircles of radius 6 cm. The length of the prism is 20 cm. The mid-point of *AD* is the origin *O*, the mid-point of *BC* is *M* and the mid-point of *DC* is *N*. The points *E* and *F* are the highest points of the semicircular ends of the prism. The point *P* lies on *EF* such that *EP* = 8 cm.

Unit vectors **i**, **j** and **k** are parallel to *OD*, *OM* and *OE* respectively.

(i) Express each of the vectors $\overrightarrow{PA}$ and $\overrightarrow{PN}$ in terms of **i**, **j** and **k**.

[3]

(ii) Use a scalar product to calculate angle *APN*.

[4]

SOLUTIONS

1. $\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ $\overrightarrow{OB} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

(i) $\overrightarrow{OA} . \overrightarrow{OB} = 8 - 9 - 2 = -3$ M1

Correct use of $a_1a_2 + b_1b_2 + c_1c_2$

$\overrightarrow{OA} . \overrightarrow{OB} = \sqrt{14} \times \sqrt{29} \cos AOB$ M1 M1

Modulus. Correct use of $ab\cos\theta$

$\rightarrow AOB = 99^o$ A14

CAO

(ii) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = 2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$ B1

CAO

Magnitude of $\overrightarrow{AB} = \sqrt{49} = 7$ M1

Use of Pythagoras + division.

$\rightarrow$ Unit vector = $\frac{1}{7}(2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k})$ A1√3

CAO (use of $\overrightarrow{BA}$ for $\overrightarrow{AB}$ has max 2/3).

(iii) $\overrightarrow{AC} = -2\mathbf{i} + 3\mathbf{j} + (p + 1)\mathbf{k}$ B1

CAO - condone **a** – **c** here.

$4 + 9 + (p + 1)^2 = 49$ M1 A1√

Correct method for forming an equation

$\rightarrow p = 5$ or -7 A14

CAO

[11]

2. (i) **OA**.**OB** = 14 – 16 – 4 = –6 M1 A1
Must be scalar from correct method.

This is –ve → Obtuse angle. B1√3
co. Correct deduction from his scalar.

(ii) **AB** = 5**i** + 10**j** – 5**k**

AX = $\frac{2}{5}$**(AB)**

OX = **OA** + **AX** M1
Needs **AB** and **OX** attempting.

OX = 4**i** – 4**j** + 2**k** A1
co

Divides by the modulus M1
Must finish with a vector, not a scalar.

Unit vector = $\frac{1}{6}$ (4**i** – 4**j** + 2**k**) A1√4
Correct for his **OX.**

[7]

3. $\overrightarrow{OP} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} 2 \\ 1 \\ q \end{pmatrix}$

(i) $\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} . \begin{pmatrix} 2 \\ 1 \\ q \end{pmatrix}$ with q = 3, = –4+3+3 =2 M1
Use of $a_1a_2+b_1b_2+c_1c_2$.

= √14.√14 cos θ = 2, cos θ = $\frac{1}{7}$ M1
Dot product linked with moduli and cos.

A13
co

(ii) $\overrightarrow{PQ}$ = **q – p** = $\begin{pmatrix} 4 \\ -2 \\ q-1 \end{pmatrix}$ M1
Allow for **p – q** or **q – p**

16 + 4 + $(q - 1)^2$ = 36 M1 A1
Use of modulus and Pythagoras

→ q = 5 or q = –3 A14
Co (for both)

[7]

4.	(i)	Vector **OD** = 4**i** + 4**j** + 5**k** One off for each error. Column vectors ok	B2, 1
		Magnitude = $\sqrt{4^2 + 4^2 + 5^2} = \sqrt{57}$ Correct use of Pythagoras.	M1
		→ Magnitude = 7.55 m Accept $\sqrt{57}$.	A14
	(ii)	Vector **OB =** 14**i** + 8**j** co	B1
		OD.OB = 4 × 14 + 4 × 8 = 88 Use of $x_1 x_2 + y_1 y_2 + z_1 z_2$ for his vectors	M1
		OD.OB = $\sqrt{57}.\ \sqrt{260}\cos\theta$ Used correctly	M1
		→ Angle *DOB* = 43.7° co	A14

[8]

5. $\mathbf{a} = \begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$.

Ok to work throughout with column vectors or with **i.j.k**

(i) **a,b =** 3 + 12 + 12 = 27 — M1
Use of $x_1x_2 + y_1y_2+z_1z_2$

a.b = √54 × √21cosθ — M1
Use of √ √cos θ

→ θ = 36.7° or 0.641 radians — A13
In either degrees or in radians.

(ii) Vector AB = **b** – α **=** $\begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}$ — M1
For use of **b – a.**

Vector OC = $\begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix} + 3\begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \\ 6 \end{pmatrix}$ — M1
For **a** + 3(**b** –**a**) or equivalent

Unit vector = Vector OC + 9. — M1
For division by Modulus of OC.

= $\frac{1}{2}$ **i** – $\frac{2}{3}$ **j** + $\frac{2}{3}$ **k** — A14
Co.

[7]

6. (i) $\overrightarrow{AB} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix}$ M1

For ± (**b** – **a**) (not **b + a**)

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$ A1

Co

Unit vector = $\frac{1}{7}\begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$ M1

Division by the modulus

A1√4

√ for his $\overrightarrow{OC}$

(ii) $m\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} + n\begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ k \end{pmatrix}$

→ $4m + 3n = 1$ and $m + 2n = 4$ M1

Forming 2 simultaneous equations

→ $m = -2$ and $n = 3$ A1

co

→ $k = -8$ M1A1 4

Equation for k in terms of m and n. co

[8]

7. (i) $\overrightarrow{PR} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ $\overrightarrow{PQ} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}$ B1

All elements of $\overrightarrow{PR}$ – any notation ok.

B2, 13

Loses one mark for each error in $\overrightarrow{PQ}$

(ii) $\overrightarrow{PQ} . \overrightarrow{PR} = –4 + 4 + 8 = 8$ M1

Must be scalar

$\left|\overrightarrow{PQ}\right| = \sqrt{24}$ $\left|\overrightarrow{PR}\right| = \sqrt{12}$ M1

As long as this is used with dot product

$\overrightarrow{PQ}$ $\overrightarrow{PR} = \sqrt{12}\ \sqrt{24} \cos QPR$ M1

Everything linked

($\overrightarrow{QP}$. $\overrightarrow{PR}$ used – still gains all M marks)

Angle QPR = 61.9° or 1.08 rad A14

Co

(iii) $\overrightarrow{QR} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$ $\left|\overrightarrow{QR}\right| = \sqrt{20}$ M1

For correct $\overrightarrow{QR}$ – cosine rule ok.

Perimeter = √12 + √24 + √20 = 12.8 cm M1 A13

Adds three roots. Co – beware fortuitous answers from incorrect sign in vectors.

[10]

8.		**OA** = 2**i** + **j** + 2**k**, **OB** = 3**i** – 2**j** + p**k**	
	(i)	(2**i** + **j** + 2**k**). (3**i** – 2**j** + p**k**) = 0 For $x_1x_2 + y_1y_2 + z_1z_2$ (in (i) or (ii))	M1
		→ $6 - 2 + 2p = 0$	
		→ $p = -2$	A12
		co	
	(ii)	(2**i** + **j** + 2**k**). (3**i** − 2**j** + 6**k**) nb Part (ii) gains 4 marks if (i) missing.	
		→ 6 – 2 + 12 allow for ± this co (M1 here if (i) not done)	A1
		$= \sqrt{9} \times \sqrt{49} \cos\theta$ All connected correctly	M1
		→ θ = 40° co	A13
	(iii)	**AB** = **i** − 3**j** + (p – 2)**k** Must be for **AB**, not **BA**.	B1
		$1^2 + 3^2 + (p-2)^2 = 3.5^2$ Pythagoras (allow if √ wrong once)	M1
		Method of solution.	DM1
		→ p = 0.5 or 3.5 co (use of **BA** can score the last 3 marks)	A14
			[9]

9.	(i)	$\overrightarrow{PA}$ = –6**i** – 8**j** – 6**k** Co – column vectors ok	B1	
		$\overrightarrow{PN}$ = 6**i** + 2**j** – 6**k** One off for each error (all incorrect sign – just one error)	B2, 1	3

(ii) $\overrightarrow{PA}.\overrightarrow{PN}$ = –36 – 16 + 36 = –16 M1

Use of $x_1x_2 + y_1y_2 + z_1z_2$

cos APN = $\frac{-16}{\sqrt{136}\sqrt{76}}$ M1

Modulus worked correctly for either one

M1

Division of "–16" by "product of moduli"

→ APN = 99° A14

Allow more accuracy

[7]