P3 Polynomial and Modulus Functions

Revision Questions

Name: ____________________

Class: ____________________

Date: ____________________

Time:	**55 minutes**
Marks:	**38 marks**
Comments:	

1.

The polynomial $x^4 + 5x + a$ is denoted by $p(x)$. It is given that $x^2 - x + 3$ is *a* factor of $p(x)$.

(i) Find the value of a and factorise $p(x)$ completely.

[6]

(ii) Hence state the number of real roots of the equation $p(x) = 0$, justifying your answer.

[2]

2. The polynomial $x^4 + 3x^2 + a$, where a is a constant, is denoted by p(x). It is given that $x^2 + x + 2$ is a factor of p(x). Find the value of a and the other quadratic factor of p(x).

[4]

3. The polynomial $4x^3 - 4x^2 + 3x + a$, where a is a constant, is denoted by $p(x)$. It is given that $p(x)$ is divisible by $2x^2 - 3x + 3$.

(i) Find the value of a.

[3]

(ii) When a has this value, solve the inequality $p(x) < 0$, justifying your answer.

[3]

4. The polynomial $x^3 - 2x + a$, where a is a constant, is denoted by $p(x)$. It is given that $(x + 2)$ is a factor of $p(x)$.

(i) Find the value of a.

[2]

(ii) When a has this value, find the quadratic factor of $p(x)$.

[2]

5. Given that a is a positive constant, solve the inequality

$$|x - 3a| > |x - a|.$$

[4]

6. Solve the inequality $|x - 2| > 3|2x + 1|$.

[4]

7. Solve the inequality $2x > |x - 1|$.

[4]

8. Find the set of values of x satisfying the inequality $|3^x - 8| < 0.5$, giving 3 significant figures in your answer.

[4]

1. (i) **EITHER:**

Attempt division by $x^2 - x + 3$ reaching a partial quotient $x^2 + x$	B1	
Complete division and equate constant remainder to zero	M1	
Obtain answer $a = -6$	A1	

OR

Commence inspection and reach unknown factor of $x^2 - x + c$	B1	
Obtain $3c = a$ and an equation in c	M1	
Obtain answer $a = -6$	A1	
State or obtain factor by $x^2 - x - 2$	B1	
State or obtain factors $x + 2$ and $x - 1$	B1 + B1	6

(ii) State that $x^2 + x - 2 = 0$, has two (real) roots B1

Show that $x^2 - x + 3 = 0$, has no (real) roots B12

[8]

2. ***EITHER:***

Attempt division by $x^2 + x + 2$ reaching a partial quotient of $x^2 + kx$	M1	
Complete the division and obtain quotient $x^2 - x + 2$		A1
Equate constant remainder to zero and solve for a		M1
Obtain answer $a = 4$		A1

OR:

Calling the unknown factor $x^2 + bx + c$, obtain an equation in b and/or c or state without working two coefficients with the correct moduli	M1
Obtain factor $x^2 - x + 2$	A1
Use $a = 2c$ to find a	M1
Obtain answer $a = 4$	A14

[4]

3. (i) ***EITHER:***

Attempt division by $2x^2 - 3x + 3$ and state partial quotient $2x$	B1
Complete division and form an equation for a	M1
Obtain $a = 3$	A1
OR1:	
By inspection or using an unknown factor $bx + c$, obtain $b = 2$	B1
Complete the factorisation and obtain a	M1
Obtain $a = 3$	A1
OR2:	
Find a complex root of $2x^2 - 3x + 3 = 0$ and substitute it in $p(x)$	M1
Equate a correct expression to zero	A1
Obtain $a = 3$	A1
OR3:	
Use $2x^2 \equiv 3x - 3$ in $p(x)$ at least once	B1
Reduce the expression to the form $a + c = 0$, or equivalent	M1
Obtain $a = 3$	A1 3

(ii)

State answer $x < -\frac{1}{2}$ only	B1
Carry out a complete method for showing $2x^2 - 3x + 3$ is never zero	M1
Complete the justification of the answer by showing that $2x^2 - 3x + 3 > 0$ for all x	A1 3

[These last two marks are independent of the B mark, so B0M1A1 is possible. Alternative methods include (a) Complete the square M1 and use a correct completion to justify the answer A1; (b) Draw a recognizable graph of $y = 2x^2 + 3x - 3$ or $p(x)$ M1 and use a correct graph to justify the answer A1; (c) Find the x-coordinate of the stationary point of $y = 2x^2 + 3x - 3$ and either find its y-coordinate or determine its nature M1, then use minimum point with correct coordinates to justify the answer A1.]

[Do not accept ≤ for <]

[6]

4. (i) Substitute $x = -2$ and equate to zero, or divide by $x + 2$ and equate constant reminder to zero, or use a factor $Ax^2 + Bx + C$ and reach an equation in a — M1

Obtain answer $a = 4$ — A12

(ii) Attempt to find quadratic factor by division or inspection — M1

State or exhibit quadratic factor $x^2 - 2x + 2$ — A12

[The M1 is earned if division reaches a partial quotient $x^2 + kx$, or if inspection has an unknown factor $x^2 + bx + c$ and an equation in b and/or c, or if inspection without working states two coefficients with the correct moduli.]

[4]

5. **EITHER:**

State or imply non-modular inequality $(x - 3a)^2 > (x - a)^2$, or corresponding equation — B1

Expand and solve the inequality, or equivalent — M1

Obtain critical value $2a$ — A1

State correct answer $x < 2a$ only — A1

OR:

State a correct linear equation for the critical value, e.g. $x - 3a = -(x - a)$, or corresponding inequality — B1

Solve the linear equation for x, or equivalent — M1

Obtain critical value $2a$ — A1

State correct answer $x < 2a$ only — A1

OR:

Make recognizable sketches of both $y = |x - 3a|$ and $y = |x - a|$ on a single diagram — B1

Obtain a critical value from the intersection of the graphs — M1

Obtain critical value $2a$ — A1

Obtain correct answer $x < 2a$ only — A1

[4]

6. **EITHER**

State or imply non-modular inequality $(x - 2)^2 > (3(2x + 1))^2$, or corresponding quadratic equation, or pair of linear equations $(x - 2) = \pm 3(2x + 1)$ B1

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1

Obtain critical values $x = -1$ and $x = -\frac{1}{7}$ A1

State answer $-1 < x < -\frac{1}{7}$ A1

OR

Obtain the critical value $x = -1$ from a graphical method, or by inspection, or by solving a linear equation or inequality B1

Obtain the critical value $x = -\frac{1}{7}$ similarly B2

State answer $-1 < x < -\frac{1}{7}$ B1

[Do not condone $\le$ for $<$; accept $-\frac{5}{35}$ and -0.14 for $-\frac{1}{7}$.]

[4]

7. ***EITHER:***

State or imply non-modular inequality $(2x)^2 > (x - 1)^2$, or corresponding equation — B1

Expand and make a reasonable solution attempt at a 2- or 3- term quadratic — M1

Obtain critical value $x = \frac{1}{3}$ — A1

State answer $x > \frac{1}{3}$ only — A1

OR:

State the relevant critical linear equation, i.e. $2x = 1 - x$ — B1

Obtain critical value $x = \frac{1}{3}$ — B1

State answer $x > \frac{1}{3}$ — B1

State or imply by omission that no other answer exists — B1

OR:

Obtain the critical value $x = \frac{1}{3}$ from a graphical method, or by inspection, or by solving a linear inequality — B2

State answer $x > \frac{1}{3}$ — B1

State or imply by omission that no other answer exists — B1

[4]

8. ***EITHER:***

State or imply non-modular inequality $-0.5 < 3^x - 8 < 0.5$, or $(3^x - 8)^2 < (0.5)^2$, or corresponding pair of linear equation or quadratic equation — B1

Use correct method for solving an equation of the form $3^x = a$, where $a > 0$ — M1

Obtain critical values 1.83 and 1.95, or exact equivalents — A1

State correct answer $1.83 < x < 1.95$ — A1

OR:

Use correct method for solving an equation of the form $3^x = a$, where $a> 0$ — M1

Obtain one critical value, e.g. 1.95, or exact equivalent — A1

Obtain the other critical value 1.83, or exact equivalent — A1

State correct answer $1.83 < x < 1.95$ — A1

[Do not condone ≤ for <, Allow final answer given in the form $1.83 < x$, (and) $x < 1.95$.]

[Exact equivalents must be i*a* terms of In or logarithms to base 10]

[SR: Solutions given as logarithms to base 3 can only earn M1 and B1 of the first scheme.]

[4]