P3 Exponential and Logarithm

Revision Questions

Name: ______________

Class: ______________

Date: ______________

Time: **109 minutes**

Marks: **93 marks**

Comments:

Q1. (a) Write down the values of p, q and r given that:

(i) $8 = 2^p$;

(1)

(ii) $\frac{1}{8} = 2^q$;

(1)

(iii) $\sqrt{2} = 2^r$.

(1)

(b) Find the value of x for which $\sqrt{2} \times 2^x = \frac{1}{8}$.

(2)
(Total 5 marks)

Q2. (a) Write each of the following in the form $\log_a k$, where k is an integer:

(i) $\log_a 4 + \log_a 10$;

(1)

(ii) $\log_a 16 - \log_a 2$;

(1)

(iii) $3 \log_a 5$.

(1)

(b) Use logarithms to solve the equation $(1.5)^{3x} = 7.5$, giving your value of x to three decimal places.

(3)

(c) Given that $\log_2 p = m$ and $\log_8 q = n$, express pq in the form 2^y, where y is an expression in m and n.

(3)
(Total 9 marks)

Q3. (a) Write down the value of:

(i) $\log_a 1$;

(1)

(ii) $\log_a a$.

(1)

(b) Given that

$$\log_a x = \log_a 5 + \log_a 6 - \log_a 1.5$$

find the value of x.

(3)
(Total 5 marks)

Q4. (a) Given that

$$\log_a x = \log_a 16 - \log_a 2$$

write down the value of x.

(1)

(b) Given that

$$\log_a y = 2 \log_a 3 + \log_a 4 + 1$$

express y in terms of A, giving your answer in a form **not** involving logarithms.

(3)
(Total 4 marks)

Q5. (a) (i) Find the value of p for which $\sqrt{125} = 5^p$.

(2)

(ii) Hence solve the equation $5^{2x} = \sqrt{125}$.

(1)

(b) Use logarithms to solve the equation $3^{2x-1} = 0.05$, giving your value of x to four decimal places.

(3)

(c) It is given that

$$\log_a x = 2(\log_a 3 + \log_a 2) - 1$$

Express x in terms of a, giving your answer in a form not involving logarithms.

(4)
(Total 10 marks)

Q6. (a) Given that $2 \ln x = 5$, find the exact value of x.

(1)

(b) Solve the equation

$$2 \ln x + \frac{15}{\ln x} = 11$$

giving your answers as exact values of x.

(5)
(Total 6 marks)

Q7. (a) Given that $3e^x = 4$, find the exact value of x.

(2)

(b) (i) By substituting $y = e^x$, show that the equation $3e^x + 20e^{-x} = 19$ can be written as $3y^2 - 19y + 20 = 0$.

(1)

(ii) Hence solve the equation $3e^x + 20e^{-x} = 19$, giving your answers as exact values.

(3)
(Total 6 marks)

Q8. (a) Given that $2 \log_k x - \log_k 5 = 1$, express k in terms of x. Give your answer in a form not involving logarithms.

(4)

(b) Given that $\log_a y = \frac{3}{2}$ and that $\log_4 a = b + 2$, show that $y = 2^p$, where p is an expression in terms of b.

(3)
(Total 7 marks)

Q9. (a) Find the value of x in each of the following:

(i) $\log_9 x = 0$;

(1)

(ii) $\log_9 x = \frac{1}{2}$.

(1)

(b) Given that

$$2 \log_a n = \log_a 18 + \log_a (n - 4)$$

find the possible values of n.

(5)
(Total 7 marks)

Q10.(a) Given that $\log_a b = c$, express b in terms of a and c.

(1)

(b) By forming a quadratic equation, show that there is only one value of x which satisfies the equation $2 \log_2(x + 7) - \log_2(x + 5) = 3$.

(6)
(Total 7 marks)

Q11. Find the value of $\log_a(a^3) + \log_a\left(\frac{1}{a}\right)$

(Total 2 marks)

Q12. Given that

$$\log_a N - \log_a x = \frac{3}{2}$$

express x in terms of a and N, giving your answer in a form not involving logarithms.

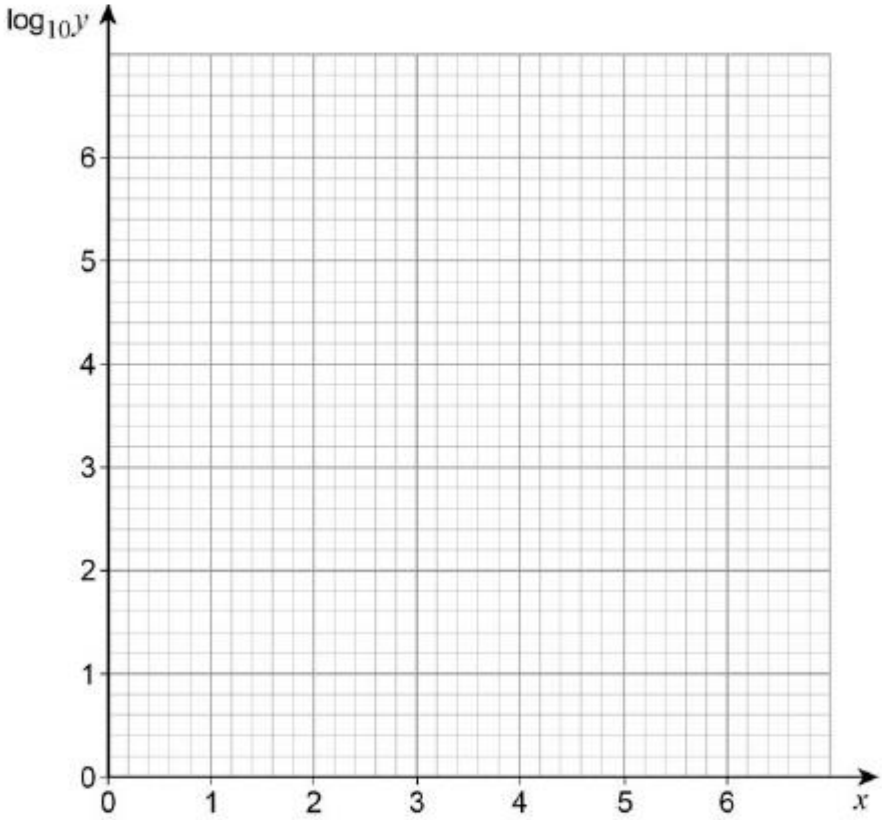
(Total 3 marks)

Q13. A student conducts an experiment and records the following data for two variables, x and y .

x	1	2	3	4	5	6
y	14	45	130	1100	1300	3400
$\log_{10} y$						

The student is told that the relationship between x and y can be modelled by an equation of the form $y = kb^x$

(a) Plot values of $\log_{10} y$ against x on the grid below.



(2)

(b) State, with a reason, which value of y is likely to have been recorded incorrectly.

(1)

(c) By drawing an appropriate straight line, find the values of k and b, giving your answers to two significant figures.

(4)
(Total 7 marks)

Q14. The variables y and x are related by an equation of the form

$$y = ax^n$$

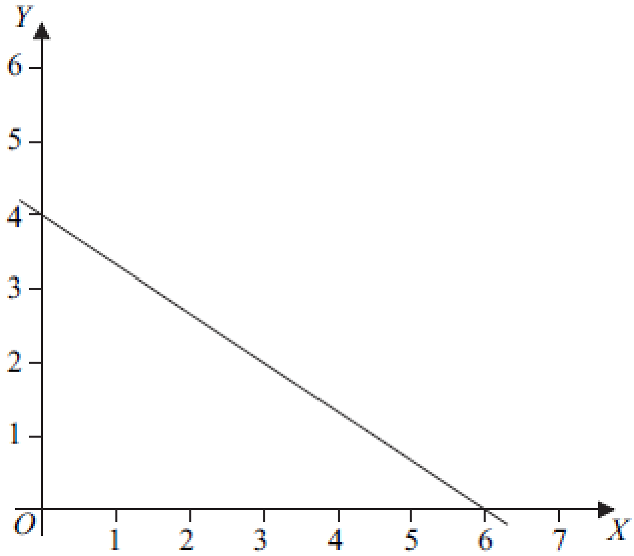
where a and n are constants.

Let $Y = \log_{10} y$ and $X = \log_{10} x$.

(a) Show that there is a linear relationship between Y and X.

(3)

(b) The graph of Y against X is shown in the diagram.



Find the value of n and the value of a.

(4)
(Total 7 marks)

Q15. David has been investigating the population of rabbits on an island during a three-year period.

Based on data that he has collected, David decides to model the population of rabbits, R, by the formula

$$R = 50e^{0.5t}$$

where t is the time in years after 1 January 2016.

(a) Using David's model:

(i) state the population of rabbits on the island on 1 January 2016;

(1)

(ii) predict the population of rabbits on 1 January 2021.

(1)

(b) Use David's model to find the value of t when R = 150, giving your answer to three significant figures.

(2)

(c) Give **one** reason why David's model may **not** be appropriate.

(1)

(d) On the same island, the population of crickets, C, can be modelled by the formula

$$C = 1000e^{0.1t}$$

where t is the time in years after 1 January 2016.

Using the two models, find the year during which the population of rabbits first exceeds the population of crickets.

(3)
(Total 8 marks)

Solutions

M1. (a) (i) (p =) 3

B1 **1**

(ii) (q =) –3

If not correct, ft on $-p$

B1F **1**

(iii) $(r =)\frac{1}{2}$

OE

B1 **1**

(b) $2^{\frac{1}{2}} \times 2^{x} = 2^{-3} \Rightarrow 2^{\frac{1}{2}+x} = 2^{-3}$

Using a law of indices or logs correctly to combine at least two of the powers of 2 PI

M1

$\Rightarrow x = -3\frac{1}{2}$

If not correct, ft on $x = q - r$ *provided method shown*

A1F **2**

[5]

M2. (a) (i) $\log_a 40$

Accept 'k = 40'

B1 **1**

(ii) $\log_a 8$

Accept 'k = 8'

B1 **1**

(iii) $\log_a 125$

Accept 'k = 125' but not 'k = 5^3*'*

B1 **1**

(b) $\log_{10} \{(1.5)^{3x}] = \log_{10} 7.5$

Correct statement having taken logs of both sides of $(1.5)^{3x} = 7.5$ OE PI or $3x = \log_{1.5} 7.5$ seen

M1

$3x \log_{10} 1.5 = \log_{10} 7.5$

$\log 1.5^{3x} = 3x \log 1.5$ OE

m1

$x = \dfrac{\lg 7.5}{3 \lg 1.5} = 1.65645\ldots = 1.656$ to 3 dp

Both method marks must have been awarded with clear use of logarithms seen

A1

3

(c) $\log_2 p = m \Rightarrow p = 2^m$; $\log_8 q = n \Rightarrow q = 8^n$

Either $p = 2^m$ or $q = 8^n$ seen or used

M1

$p = 2^m$ and $q = 2^{3n}$

Writing $8^n = 2^{3n}$ and having $p = 2^m$

m1

$pq = 2^m \times (2^3)^n = 2^m \times 2^{3n}$ so $pq = 2^{m+3n}$

Accept $y = m + 3n$

A1

3

M3. (a) (i) $\log_a 1 = 0$

B1

1

(ii) $\log_a a = 1$

B1

1

(b) $\log_a x = \log_a (5 \times 6) - \log_a 1.5$

One law of logs used correctly

M1

$\log_a x = \log_a \left(\frac{5 \times 6}{1.5}\right)$

A second law of logs used correctly

M1

$\log_a x = \log_a 20 \Rightarrow x = 20$

A1

3

[5]

M4. (a) $x = 8$

No clear log law errors seen.

Condone answer left as $\frac{16}{2}$

B1

1

(b) $\log_a y = \log_a 3^2 + \log_a 4 + 1$

One law of logs used correctly

M1

$\log_a y = \log_a (3^2 \times 4) + 1$

Either a second law of logs used correctly
or the 1 written as $\log_a a$

M1

$\log_a y = \log_a (3^2 \times 4) + \log_a a = \log_a 36a$
$\Rightarrow y = 36a$

CSO

A1

3

[4]

M5. (a) (i) $\sqrt{125} = \sqrt{25 \times 5} = 5\sqrt{5}$

OE eg $\sqrt{125} = \sqrt{5^3}$ *or* $5^{1.5}$ *seen*

M1

$5^p = \sqrt{125} \Rightarrow p = 1.5$

Correct value of p must be explicitly stated

A1

Alternative:

$p \log 5 = \frac{1}{2} \log 125$

2

OE eg p log 5 = log 11.18

or eg $p = \log_5 \sqrt{125}$

(M1)

$p \log 5 = \frac{3}{2} \log 5 \Rightarrow p = \frac{3}{2}$

Correct value of p must be explicitly stated

(A1)

(ii) $5^{2x} = \sqrt{125} = 5^p \Rightarrow x = 0.5p = 0.75$

Must be 0.5 × c's value of p

SC: x = 0.75 with c's ans (a)(i) $5^{1.5}$ *scores B1F*

B1F

1

(b) $3^{2x-1} = 0.05$

$(2x - 1)\log 3 = \log 0.05$

Take logs of both sides and use 3rd law of logs. PI eg by $2x - 1 = \log_3 0.05$ *seen*

M1

$x = \frac{\log_{10} 0.05}{2\log_{10} 3} + \frac{1}{2}$

Correct rearrangement to x = ... PI

m1

= –0.8634(165...) = –0.8634 to 4dp

Condone > 4 dp. Must see logs clearly ***used*** *in solution, so NMS scores 0/3*

A1

3

(c) $\log_a x = 2(\log_a 3 + \log_a 2) - 1$

$= 2 \log_a (3 \times 2) - 1$

A valid law of logs used

M1

$= \log_a (6^2) - 1$

= $\log_a 36 - \log_a a$

$\log_a a = 1$ quoted or used

or $\log_a \frac{x}{k} = -1 \Rightarrow \frac{x}{k} = a^{-1}$ *OE*

B1

$\log_a x = \log_a\left(\frac{36}{a}\right) \Rightarrow x = \frac{36}{a}$

CSO Must be $x = \frac{36}{a}$ *or* $x = 36a^{-1}$

A1

4

[10]

M6. (a) $2 \ln x = 5$

$\ln x = \frac{5}{2}$ $x = e^{\frac{5}{2}}$

B1

1

(b) $2\ln x + \frac{15}{\ln x} = 11$

$2(\ln x)^2 - 11 \ln x + 15 = 0$

Forming quadratic equation in ln x
condone poor notation

M1

$(2 \ln x - 5)(\ln x - 3) = 0$

Attempt at factorisation/formula

m1

$\ln x = \frac{5}{2}, 3$ condone $2 \ln x = 5$

A1

$x = e^{\frac{5}{2}}, e^3$

[SC for substituting $x = e^{\frac{5}{2}}$ *or equivalent*

into equation and verifying B1 $\left(\frac{1}{5}\right)$*]*

A1, A1

5

[6]

M7. (a) $3e^x = 4$

$e^x = \frac{4}{3}$

M1

$x = \ln \frac{4}{3}$

A1

2

(b) (i) $3e^x + 20e^{-x} = 19$

$3y + \frac{20}{y} = 19$ or $3e^{2x} + 20 = 19e^x$

$3y^2 - 19y + 20 = 0$

AG

B1

1

(ii) $(3y - 4)(y - 5) = 0$

$y = \frac{4}{3}, 5$

B1

$\therefore x = \ln\frac{4}{3}, \ln 5$

ln (their + ve y's)

M1A1

3

[6]

M8. (a) $\log_k x^2 - \log_k 5 = 1$

A valid law of logs used correctly

M1

$\log_k \frac{x^2}{5} = 1$

Another valid law of logs used correctly
***or** correct method to reach log f(x) = log 5k*

M1

$\log_k \frac{x^2}{5} = \log_k k$ [or $\log x^2 = \log 5k$]

PI by next line

A1

$\Rightarrow \frac{x^2}{5} = k$ ie $k = \frac{x^2}{5}$

Accept either of these two forms.

A1

4

(b) $\log_a y = \frac{3}{2}$ $\log_4 a = b + 2$

$\Rightarrow y = a^{\frac{3}{2}}$ $\Rightarrow a = 4^{b+2}$

For either equation

M1

$y = \left(4^{b+2}\right)^{\frac{3}{2}}$

Elimination of a from two correct equations not involving logarithms

m1

$y = 2^{3(b+2)}$; $y = 2^{3b+6}$

CSO Either form acceptable

A1

3

[7]

M9. (a) (i) (x =) 1

CAO

B1

1

(ii) (x =) 3

CAO

B1

1

(b) $\log_a n^2 = \log_a 18(n - 4)$

A valid law of logs applied to correct logs

A second valid law of logs applied to correct logs

$n^2 - 18n + 72 = 0$

ACF of these terms eg $n^2 - 18n = -72$

A1

$(n - 6)(n - 12) = 0$

Valid method to solve quadratic, dep on both the previous Ms

m1

$n = 6$, $n = 12$

Both values required
***SC** NMS max (out of 5) B3 for both 6 and 12 without uniqueness considered; max B1 for either 6 or 12 only*

A1

5

[7]

M10.(a) $b = a^c$

B1

1

(b) $2 \log_2 (x + 7) - \log_2 (x + 5) = 3$

$\log_2 (x + 7)^2 - \log_2 (x + 5) = 3$

A law of logs used correctly on a correct expression.

M1

$\log_2 \frac{(x+7)^2}{x+5} = 3$

A further correct use of law of logs on a correct expression.

M1

$= 3 \log_2 2 = \log_2 2^3$

$\Rightarrow \frac{(x+7)^2}{x+5} = 2^3$

$3 = 3 \log_2 2$ or $3 = \log_2 2^3$ (= $\log_2 8$) seen
or
eg log f(x) = 3 $\Rightarrow$ f(x) = 2^3 (= 8) OE

B1

$\Rightarrow (x + 7)^2 = 8(x + 5)$

Correct equation having eliminated logs and fractions

A1

$\Rightarrow x^2 + 14x + 49 = 8x + 40$

$\Rightarrow x^2 + 6x + 9$ (= 0)

A1

Since $6^2 - 4(1)(9) = 0$, (there is only) one value of x (which satisfies the given equation).

A1

6

[7]

M11.

Marking Instructions	AO	Marks	Typical Solution
Correctly applies a single law of logs with either term	AO1.1a	M1	$\log_a(a^3) + \log_a\left(\frac{1}{a}\right) = 3 + (-1)$
States correct final answer (NMS scores full marks)	AO1.1b	A1	$= 3 - 1$ $= 2$
			Total 2 marks

M12. $\log_a N - \log_a x = \frac{3}{2}$

$\log_a \frac{N}{x} = \frac{3}{2}$

A log law used correctly. PI by next line.

M1

$\frac{N}{x} = a^{\frac{3}{2}}$

Logarithm(s) eliminated correctly

m1

$x = a^{-\frac{3}{2}} N$

ACF of RHS

A1

[3]

M13.

	Marking Instructions	AO	Marks	Typical Solution
(a)	Obtains (at least four) correct $\log_{10} y$ values, in table or plotted	AO1.1a	M1	(1, 1.1) (2, 1.7) (3, 2.1) (4, 3.0) (5, 3.1) (6, 3.5)
	Plots **all** points correctly	AO1.1b	A1	(Points above plotted on grid)

(b)	Identifies y = 1100 and gives correct reason	AO2.2b	B1	(4, 1100), as it is not on the line that the other points are close to
(c)	Uses laws of logs. (May earn in part **(a)** if laws of logs were used there)	AO1.1a	M1	$\log_{10} y = \log_{10} k + x\log_{10} b$ Vertical intercept c = 0.68 (=$\log_{10} k$) Therefore from intercept: $k = 10^{0.68}$ Gradient m = 0.48 =$\log_{10} b$ Therefore from gradient: $b = 10^{0.48}$
	Draws straight line **And** calculates/measures the vertical intercept c and attempts 10^c **or** calculates/measures gradient m and attempts 10^m Alternatively uses regression line from calculator to get intercept and gradient	AO1.1a	M1	
	Finds correct value of k or correct value of b − any accuracy	AO1.1b	A1	k = 4.8
	Finds correct values for both k and b given to 2 sig figs	AO1.1b	A1	b = 3.0
				Total 7 marks

M14.(a) $y = ax^n \Rightarrow \log_{10} y = \log_{10} ax^n$

$\log_{10} y = \log_{10} a + \log_{10} x^n$

Take logs and apply one log law in soln. correctly PI.

M1

$\log_{10} y = \log_{10} a + n\log_{10} x$

Apply a further log law correctly.

m1

$Y = \log_{10} a + nX$ (which is a linear relationship between Y and X.)

Correct eqn. with base 10 (or lg or later evidence of use of base 10 if log without base here)

A1

(b) n = gradient of line

Stated or used.

Accept $n = \pm\frac{2}{3}$ OE as evidence

M1

$n = -\frac{2}{3}$

$n = -\frac{2}{3}$ *(OE 3sf)*

A1

$\log_{10} a = 4$

Equating c's constant term [must involve a log] in
c's (a) eqn. to the Y-intercept value PI by correct value of a

M1

$a = 10^4$ (=10 000)

A1

4

[7]

M15.

	Marking Instructions	AO	Marks	Typical Solution
(a) (i)	States correct value CAO	AO3.4	B1	50
(ii)	States correct integer value CAO	AO3.4	B1	609
(b)	Forms correct equation and rearranges to obtain $e^{0.5t} = \ldots$	AO3.4	M1	$150 = 50e^{0.5t}$ so $e^{0.5t} = 3$
	Obtains the correct solution. Must give answer to 3 sf	AO1.1b	A1	$t = 2\ln 3 = 2.20$
(c)	1 mark for any clear valid reason, must be set in context of the question	AO3.5b	E1	No constraint on the number of rabbits (ie could go off to infinity) **OR** Model is only based on the 3 years of the study. Things may change **OR** Continuous model but number of rabbits is discrete **OR** Ignores extraneous factors such as disease, predation, limited food supply
(d)	Forms an equation with exponentials by letting $R = C$ PI	AO3.4	M1	$1000e^{0.1t} = 50e^{0.5t}$ $20 = e^{0.4t}$
	Solves 'their' equation correctly	AO1.1a	M1	$t = \ln 20 \div 0.4$ $= 7.49$
	States correct answer as the year 2023 CAO NMS scores full marks for 2023	AO3.2a	A1	2023
				Total 8 marks