P3 Trig Revision

Revision Questions

Name: ____________________

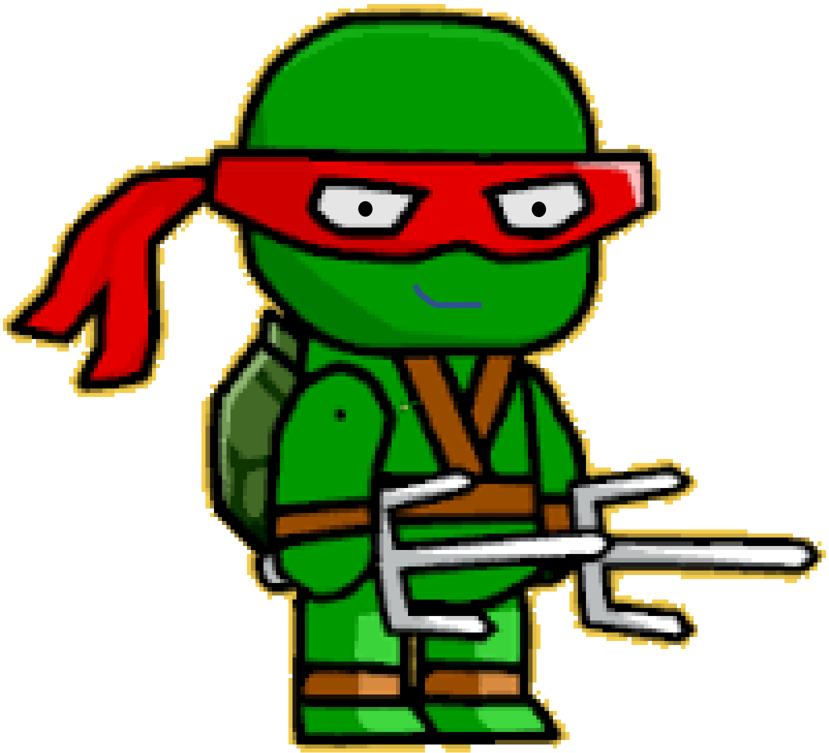
Class: ____________________

Date: ____________________

Time: **132 minutes**

Marks: **110 marks**

Comments:



Q1. Prove the identity $\cot^2\theta - \cos^2\theta \equiv \cot^2\theta\cos^2\theta$

(Total 3 marks)

Q2. (a) (i) Express sin 2θ *a*nd cos 2θ in terms of sin θ and cos θ.

(2)

(ii) Given that $0 < \theta < \frac{\pi}{2}$ and cos $\theta = \frac{3}{5}$, show that sin $2\theta = \frac{24}{25}$ and find thevalue of cos 2θ.

(2)

(b) A curve has parametric equations

$$x = 3 \sin 2\theta, \; y = 4 \cos 2\theta$$

(i) Find $\frac{dy}{dx}$ in terms of θ.

(3)

(ii) At the point P on the curve, cos $\theta = \frac{3}{5}$ and $0 < \theta < \frac{\pi}{2}$. Find an equation of the tangent to the curve at the point P.

(3)

(Total 10 marks)

Q3. (a) Solve the equation

$$\text{cosec } x = 3$$

giving all values of x in radians to two decimal places, in the interval $0 \leq x \leq 2\pi$.

(2)

(b) By using a suitable trigonometric identity, solve the equation

$$\cot^2 x = 11 - \text{cosec } x$$

giving all values of x in radians to two decimal places, in the interval $0 \leq x \leq 2\pi$.

(6)
(Total 8 marks)

Q4. (a) Express $\cos x + 3 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
Give your value of α, in radians, to three decimal places.

(3)

(b) (i) Hence write down the minimum value of $\cos x + 3 \sin x$.

(1)

(ii) Find the value of x in the interval $0 \leq x \leq 2\pi$ at which this minimum occurs, giving your answer, in radians, to three decimal places.

(2)

(c) Solve the equation $\cos x + 3 \sin x = 2$ in the interval $0 \leq x \leq 2\pi$, giving all solutions, in radians, to three decimal places.

(4)

(Total 10 marks)

Q5. (a) (i) Given that $\tan 2x + \tan x = 0$, show that $\tan x = 0$ or $\tan^2 x = 3$.

(3)

(ii) Hence find all solutions of $\tan 2x + \tan x = 0$ in the interval $0° < x < 180°$.

(1)

(b) (i) Given that $\cos x \neq 0$, show that the equation

$$\sin 2x = \cos x \cos 2x$$

can be written in the form

$$2 \sin^2 x + 2 \sin x - 1 = 0$$

(3)

(ii) Show that all solutions of the equation $2 \sin^2 x + 2 \sin x - 1 = 0$ are given by $\sin x = \frac{\sqrt{3} - 1}{p}$, where p is an integer.

(3)
(Total 10 marks)

Q6. (a) Express 2 sin x + 5 cos x in the form R sin(x + α), where R > 0 and 0° < α < 90°. Give your value of α to the nearest 0.1°.

(3)

(b) (i) Write down the maximum value of 2 sin x + 5 cos x.

(1)

(ii) Find the value of x in the interval 0° ≤ x ≤ 360° at which this maximum occurs, giving the value of x to the nearest 0.1°.

(2)
(Total 6 marks)

Q7. (a) Solve the equation sec x = –5, giving all values of x in radians to two decimal places in the interval $0 < x < 2\pi$.

(3)

(b) Show that the equation

$$\frac{\operatorname{cosec} x}{1+\operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1-\operatorname{cosec} x} = 50$$

can be written in the form

$$\sec^2 x = 25$$

(4)

(c) Hence, or otherwise, solve the equation

$$\frac{\operatorname{cosec} x}{1+\operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1-\operatorname{cosec} x} = 50$$

giving all values of x in radians to two decimal places in the interval $0 < x < 2\pi$.

(3)
(Total 10 marks)

Q8.(a) Express sin x − 3 cos x in the form R sin(x − α), where R > 0 and0° < α < 90°, giving your value of α to the nearest 0.1°.

(3)

(b) Hence find the values of x in the interval 0° < x < 360° for which

$$\sin x - 3\cos x + 2 = 0$$

giving your values of x to the nearest degree.

(4)
(Total 7 marks)

Q9.(a) Show that

$$\frac{\sec^2 x}{(\sec x + 1)(\sec x - 1)}$$

can be written as $\operatorname{cosec}^2 x$.

(3)

(b) Hence solve the equation

$$\frac{\sec^2 x}{(\sec x + 1)(\sec x - 1)} = \operatorname{cosec} x + 3$$

giving the values of x to the nearest degree in the interval $-180° < x < 180°$.

(6)

(c) Hence solve the equation

$$\frac{\sec^2(2\theta - 60°)}{(\sec(2\theta - 60°) + 1)(\sec(2\theta - 60°) - 1)} = \operatorname{cosec}(2\theta - 60°) + 3$$

giving the values of θ to the nearest degree in the interval $0° < \theta < 90°$.

(2)

(Total 11 marks)

Q10.(a) Show that the equation

$$\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = 32$$

can be written in the form

$$\operatorname{cosec}^2 \theta = 16$$

(4)

(b) Hence, or otherwise, solve the equation

$$\frac{1}{1+\cos(2x-0.6)} + \frac{1}{1-\cos(2x-0.6)} = 32$$

giving all values of x in radians to two decimal places in the interval $0 < x < \pi$.

(5)
(Total 9 marks)

Q11.(a) Use the Factor Theorem to show that $4x - 3$ is a factor of

$$16x^3 + 11x - 15$$

(2)

(b) Given that $x = \cos\theta$, show that the equation

$$27\cos\theta\cos 2\theta + 19\sin\theta\sin 2\theta - 15 = 0$$

can be written in the form

$$16x^3 + 11x - 15 = 0$$

(4)

(c) Hence show that the only solutions of the equation

$$27\cos\theta\cos 2\theta + 19\sin\theta\sin 2\theta - 15 = 0$$

are given by $\cos\theta = \frac{3}{4}$.

(4)
(Total 10 marks)

Q12. By forming and solving a quadratic equation, solve the equation

$$8 \sec x - 2 \sec^2 x = \tan^2 x - 2$$

in the interval $0 < x < 2\pi$, giving the values of x in radians to three significant figures.

(Total 7 marks)

Q13.(a) By using a suitable trigonometrical identity, solve the equation

$$\tan^2 \theta = 3(3 - \sec \theta)$$

giving all solutions to the nearest 0.1° in the interval $0° < \theta < 360°$.

(6)

(b) Hence solve the equation

$$\tan^2(4x - 10°) = 3[3 - \sec(4x - 10°)]$$

giving all solutions to the nearest 0.1° in the interval $0° < x < 90°$.

(3)
(Total 9 marks)

M1.

Marking Instructions	AO	Marks	Typical Solution
Recalls a correct trig identity, which could lead to a correct answer	AO1.2	B1	(LHS ≡) $\cot^2\theta - \cos^2\theta$ $\equiv \frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta$ $\equiv \cos^2\theta\left(\frac{1}{\sin^2\theta}\right) - 1$ $\equiv \cos^2\theta(\operatorname{cosec}^2\theta - 1)$ $\equiv \cos^2\theta\cot^2\theta$ (≡ RHS) **AG**
Performs some correct algebraic manipulation **and** uses second identity to commence proof (at least two lines of argument)	AO2.1	R1	
Concludes a rigorous mathematical argument to prove given identity **AG** Must start with one side and through clear logical steps arrive at the other side. In order to be sufficiently clear, each line should be a single step, unless clear further explanation is given.	AO2.1	R1	
			Total 3 marks

M2. (a) (i) $\sin 2\theta = 2\sin\theta\cos\theta$

B1

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

OE condone use of x etc, but variable must be consistent

B1

2

(ii) $\sin\theta = \frac{4}{5} \Rightarrow \sin 2\theta = 2 \times \frac{4}{5} \times \frac{3}{5} = \frac{24}{25}$

AG
Use of 106.26° …. B0

B1

or

$2 \times \sin\left(\cos^{-1}\frac{3}{5}\right) \times \frac{3}{5}$

$\cos 2\theta = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$

–0.28

B1

2

(b) (i) $\frac{dx}{d\theta} = 6\cos 2\theta$, $\frac{dy}{d\theta} = -8\sin 2\theta$

Attempt both derivatives. ie $p \cos 2\theta$

M1

Both correct. $q \sin 2\theta$

A1

$\frac{dy}{dx} = -\frac{4}{3}\frac{\sin 2\theta}{\cos 2\theta}$ ISW

CSO OE

A1

3

(ii) $p\left(\frac{72}{25}, -\frac{28}{25}\right)$

(2.88, –1.12)

B1F

Gradient $= -\frac{4}{3} \times -\frac{24}{7}$

Their $\frac{q \sin 2\theta}{p \cos 2\theta}$ **or** $\frac{p \cos 2\theta}{q \sin 2\theta}$

must be working with rational numbers

M1

Tangent $y + \frac{28}{25} = \frac{32}{7}\left(x - \frac{72}{25}\right)$ ISW

Any correct form.
7y = 32x – 100
Fractions in simplest form
Equation required

A1

3

[10]

M3. (a) $\sin x = \frac{1}{3}$, or sight of ±0.34, ±0.11π or ±19.47 (or better)

M1

x = 0.34, 2.8(0) AWRT

Penalise if incorrect answers in range;
ignore answers outside range

A1

2

(b) $\text{cosec}^2\, x - 1 = 11 - \text{cosec}\, x$

Correct use of $\cot^2 x = \text{cosec}^2\, x - 1$

M1

$\text{cosec}^2\, x + \text{cosec}\, x - 12\ (= 0)$

A1

$(\text{cosec}\, x + 4)(\text{cosec}\, x - 3)\ (= 0)$

Attempt at Factors
Gives cosec x or – 12 when expanded
Formula one error condoned

m1

$\left.\begin{array}{l}\text{cosec}\, x = -4, 3 \\ \sin x = -\frac{1}{4}, \frac{1}{3}\end{array}\right\}$

Either Line

A1

$\sin x = -\frac{1}{4}$

$\Rightarrow x$ = 3.39, 6.03 AWRT

3 correct or their two answers from (a)
and 3.39, 6.03

B1F

0.34, 2.8(0) AWRT

4 correct and no extras in range
ignore answers outside range
SC 19.47, 160.53, 194.48, 345.52 B1

Alternative

$$\frac{\cos^2 x}{\sin^2 x} = 11 - \frac{1}{\sin x}$$

$\cos^2 x = 11 \sin^2 x - \sin x$

Correct use of trig ratios and multiplying by 2 $\sin^2 x$

(M1)

$1 - \sin^2 x = 11 \sin^2 x - \sin x$

$0 = 12 \sin^2 x - \sin x - 1$

(A1)

$0 = (4 \sin x + 1)(3 \sin x - 1)$

Attempt at factors as above

(m1)

$$\sin x = -\frac{1}{4}, \frac{1}{3}$$

(A1)

As above

(B1F)(B1)

[8]

M4. (a) $R = \sqrt{10}$

Accept R = 3.16 or better

B1

$\tan \alpha = 3$

OE

M1

$\alpha = 1.249$ ignore extra out of range

AWRT 1.25 SC α = 0.322 B1 radians only

A1

3

(b) (i) minimum value = $-\sqrt{10}$

F on R

B1F

(ii) $\cos(x - \alpha) = -1$

M1

$x = 4.391$

AWRT 4.39
51.56° or .. .57° or better

A1F

2

(c) $\cos(x - \alpha) = \dfrac{2}{\sqrt{10}}$

M1

$x - \alpha = \pm 0.886 \quad 5.397$
ignore extra out of range

Two values, accept 2dp and condone 5.4
condone use of degrees

A1

$x = 0.36296.. \quad 2.13512..$

F on $x - \alpha$, either value. AWRT

A1F

$x = 0.363 \quad 2.135$

CSO *3dp or better*

A1

Alternative

$10 \sin^2 x - 12 \sin x + 3 = 0$

Or equivalent quadratic using cos x
(ie $\sin^2 x + \cos^2 x = 1$ used)

M1

$\sin x$ = two numerical answers

Or equivalent using cos x

A1F

$-1 \le$ ans ≤ 1
x = one correct answer

A1F

$x = 0.363 \quad 2.135$

CSO *3 dp or better*

A1

4

[10]

M5. (a) (i) $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

Condone numerator as tan x + tan x

B1

$2\tan x + \tan x\,(1 - \tan^2 x) = 0$

Multiplying throughout by their denominator

M1

$\tan x = 0$

or $(2 + 1 - \tan^2 x) = 0 \Rightarrow \tan^2 x = 3$

AG *Must show tan x = 0* ***and*** *tan²x = 3*

A1

3

Alternative

$\tan 2x = \dfrac{\sin 2x}{\cos 2x} = \dfrac{2\sin x \cos x}{\cos^2 x - \sin^2 x}$

$\dfrac{2\sin x \cos x}{\cos^2 x - \sin^2 x} + \dfrac{\sin x}{\cos x} 0$

(B1)

$2\sin x \cos^2 x + \sin x\,(\cos^2 x - \sin^2 x) = 0$

$\sin x\,(2\cos^2 x + \cos^2 x - \sin^2 x) = 0$

(M1)

$\left.\begin{array}{l}\Rightarrow \sin x = 0\\ \Rightarrow \tan x = 0\end{array}\right\}$ $\begin{array}{l}\text{and}\\ \text{and}\end{array}$ $\left.\begin{array}{l}3\cos^2 x = \sin^2 x\\ \tan^2 x = 3\end{array}\right\}$

(A1)

(3)

(ii) $x = 60$ **AND** $x = 120$

Condone extra answers outside interval
eg 0 and 180

(b) (i) $2\sin x\cos x = \cos x f(x)$

Where $f(x) = \cos^2 x - \sin^2 x$

or $2\cos^2 x - 1$ *or* $1 - 2\sin^2 x$

M1

$2\sin x\cos x = \cos x(1 - 2\sin^2 x)$

A1

$(\cos x \neq 0)$ $2\sin x = 1 - 2\sin^2 x$

$2\sin^2 x + 2\sin x - 1 = 0$

AG

A1

3

(ii) $\sin x = \dfrac{-2 \pm \sqrt{4 - 4 \times 2 \times (-1)}}{2 \times 2}$

Correct use of quadratic formula or completing the square or correct factors

M1

$\sin x = \dfrac{-2 \pm 2\sqrt{3}}{4}$

$\sqrt{12}$ *must be simplified and must have ±.*

A1

$\left.\begin{array}{l} \sin x = \dfrac{-1-\sqrt{3}}{2} \text{ has no solution} \\ \sin x = \dfrac{\sqrt{3}-1}{2} \end{array}\right\}$

Reject one solution and state correct solution.

E1

3

[10]

M6. (a) $R = \sqrt{29}$

Accept 5.4 or 5.38, 5.39, 5.385….

$R\sin\alpha = 5$ or $R\cos\alpha = 2$ or $\tan\alpha = \dfrac{5}{2}$

$\alpha = 68.2°$

Condone $\alpha = 68.20°$

A1

(b) (i) (maximum value =) $\sqrt{29}$

ft on R

B1ft

1

(ii) $\sin(x + \alpha) = 1$

Or $x + \alpha = 90, x + \alpha = \frac{\pi}{2}$

M1

x = 21.8° only

No ISW

A1

[6]

M7. (a) cos x = –0.2

Or tan x =(±) $\sqrt{24}$

M1

x = 1.77, 4.51 AWRT

One correct value

A1

Second correct value and no extra values in interval 0 to 6.28…

Ignore answers outside interval

SC

x = 1.8, 4.5 with or without working M1 A1 A0

SC (using degrees)

101.54, 281.54 M1 A1 A0

101.5, 281.5 M1 A0 A0

SC

No working shown

2 correct answers 3/3

1 correct answer 2/3

A1

3

(b) LHS

$$= \frac{\text{cosec}x(1-\text{cosec}x)-\text{cosec}x(1+\text{cosec}x)}{(1+\text{cosec}x)(1-\text{cosec}x)}$$

Correctly combining fractions but condone poor use, or omission, of brackets

M1

$$= \frac{\text{cosec}\,x-\text{cosec}^2 x-\text{cosec}\,x-\text{cosec}^2 x}{1-\text{cosec}^2 x}$$

Allow recovery from incorrect brackets

A1

$$= \frac{-2\,\text{cosec}^2 x}{-\cot^2 x} \text{ or } \frac{-2(1+\cot^2 x)}{-\cot^2 x}$$

Correct use of relevant trig identity
eg $\text{cosec}^2 x = 1 + \cot^2 x$

m1

$2\sec^2 x = 50$

$\sec^2 x = 25$ AG

All correct with no errors seen INCLUDING correct brackets on 1st line

A1

Or

$$\frac{\text{cosec}x}{1+\text{cosec}x}-\frac{\text{cosec}x}{1-\text{cosec}x}=50$$

$\text{cosec}\,x\,(1 - \text{cosec}\,x) - \text{cosec}\,x\,(1 + \text{cosec}\,x)$
$= 50\,(1 + \text{cosec}\,x)(1 - \text{cosec}\,x)$

Correctly eliminating fractions but condone poor use, or omission, of brackets

(M1)

$\text{cosec}\,x - \text{cosec}^2 x - \text{cosec}\,x - \text{cosec}^2 x$
$= 50\,(1 - \text{cosec}^2 x)$

Allow recovery from incorrect brackets

(A1)

$48\text{cosec}^2\, x = 50$

$\sin^2 x = \frac{24}{25} \Rightarrow \cos^2 x = \frac{1}{25}$

Correct use of relevant trig identity

eg $\sin^2 x = 1 - \cos^2 x$

(m1)

$\sec^2 x = 25$ AG

All correct with no errors seen INCLUDING correct brackets on 1st line

(A1)

(c) $\sec x = \pm 5$

Or $\cos x = \pm 0.2$

Or $\tan x = \pm\sqrt{24}$

M1

x = 1.77, 4.51, 1.37, 4.91 (AWRT)

3 correct

A1

4 correct and no other answers in interval

Ignore answers outside interval

SC

1.8, 4.5, 1.4, 4.9

With or without working *M1 A1*

SC

their 2 answers from (a)

+1.37, 4.91 *(AWRT)* *2/3*

SC For this part, if in degrees

max mark is *M1 A0*

SC

No working shown

4 correct answers *3/3*

3 correct answers *2/3*

0, 1, 2 correct answers 0/3

A1

3

[10]

M8.(a) $R = \sqrt{10}$

Accept 3.2 or better. Can be earned in (b)

B1

$\tan \alpha = 3$

OE; M0 if tan α = −3 seen

M1

$\alpha = 71.6$ or better

α = 71.56505…

A1

3

(b) $\sin(x \pm \alpha) = \frac{-2}{R}$

or their R and / or their α; PI

M1

$x(= -39.2 + 71.6) = 32(.333)$

32 or better
Condone 32.4

A1

or

$x - 71.6 = 219.2$

must see 219 and 72 or better PI by 291 or better as answer
Condone extra solutions

m1

$x = 291$

Condone 290.8 or better CSO Withhold final A1 if more than two answers given within interval

A1

4

[7]

M9.(a) $\frac{\sec^2 x}{(\sec x + 1)(\sec x - 1)} = \frac{\sec^2 x}{\sec^2 x - 1}$

$\sec^2 x = 1 + \tan^2 x$ used

M1 for correct use of $\sec^2 x = 1 + \tan^2 x$ at least once or ($\text{cosec}^2 x = 1 + \cot^2 x$)

M1

$= \frac{\sec^2 x}{\tan^2 x}$ or $\frac{1+\tan^2 x}{\tan^2 x}$

$\left(= \frac{1}{\cos^2 x \tan^2 x}\right)$

$= \frac{1}{\sin^2 x}$ or $\cot^2 x + 1$

Shown, with no errors

A1

$= \text{cosec}^2\, x$

AG (No errors, omissions or poor notations seen)

A1

3

(b) $\text{cosec}^2\, x = \text{cosec}\, x + 3$

$\text{cosec}^2\, x - \text{cosec}\, x - 3 = 0$

must have = 0

correct solution of the quadratic, or by completing the square

B1

$\text{cosec}\, x = \frac{1 \pm \sqrt{13}}{2}$ or (2.3… **and** − 1.3…)

$\left(\text{cosec}\, x = \pm\sqrt{\frac{13}{4}} + \frac{1}{2}\right)$

PI by values for sin x

M1

$\sin x = \frac{2}{1 \pm \sqrt{13}}$

B1F for cosec x = $\frac{1}{\sin^2 x}$ *seen or implied*

B1F

= 0.434 **and** − 0.768 (or −0.767)

PI

A1

B1 for any three values correct AWRT

B1

x = 26°,154°, − 50°,−130°

B1 for all four values correct AWRT and no extras in the interval −180 < x <180°

B

(c) $2\theta - 60° = x$

where x is a written value from candidate's (b) in degrees
PI by their answer

M1

$\theta = 43°, 5°$

CSO

A1

Ignore solutions outside interval $0° < \theta < 90°$

2

[11]

M10.(a) LHS = $\frac{(1-\cos\theta)+(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

Combining fractions

M1

$= \frac{2}{1-\cos^2\theta}$

Correctly simplified

A1

$= \frac{2}{\sin^2\theta}$

Use of $\sin^2\theta + \cos^2\theta = 1$

m1

$2\ \text{cosec}^2\theta = 32$
$\text{cosec}^2\theta = 16$

AG; no errors seen

A1

OR

$1 - \cos\theta + 1 + \cos\theta = 32(1 + \cos\theta)(1 - \cos\theta)$ (M1)
$2 = 32(1 - \cos^2\theta)$ (A1)
$2 = 32\sin^2\theta$ (m1)
$\text{cosec}^2\theta = 16$ (A1)

4

(b) $\text{cosec}\ y = (\pm)\sqrt{16}$ or better (PI by further working)

or $\sin y = (\pm)\sqrt{\frac{1}{16}}$ or better

M1

(y =)
0.253, (2.889,) (3.394,) (6.031,) (−0.253)

Sight of any of these correct to 3dp or better

B1

(y =)
0.25, 2.89, 3.39 (or better)

Must see these 3 answers, with or without either / both of −0.25 or 6.03
Ignore answers outside interval −0.25 to 6.03 but extras in this interval scores A0

A1

x = 0.43, 1.74, 2(.00), 0.17

3 correct (must be 2 dp)

All 4 correct (must be 2 dp) and no extras in interval (ignore answers outside interval)

B1
B1

5

[9]

M11.(a) $16\left(\frac{3}{4}\right)^3 + 11\left(\frac{3}{4}\right) - 15$

Evaluate f $\left(\frac{3}{4}\right)$ *not long division.*

M1

$= \frac{27}{4} + \frac{33}{4}$ − 15 = 0 ⇒ factor

Processing and conclusion.

A1

For A1; minimum processing seen; $16 \times \frac{27}{64} + 11 \times \frac{3}{4} - 15 = 0$ *; 15 − 15 = 0*

and no other working is A0 minimum conclusion = 0 hence factor

2

(b) 27cosθ(2cos$^2\theta$ − 1) +

Use acf of cos 2θ formula

B1

19sinθ(2sinθcosθ) − 15 = 0

Use acf of sin 2θ formula

$54\cos^3\theta - 27\cos\theta + 38(1 - \cos^2\theta)\cos\theta - 15 = 0$

All in cosines.

M1

$16\cos^3\theta + 11\cos\theta - 15 = 0$

$x = \cos\theta \Rightarrow 16x^3 + 11x - 15 = 0$

Simplification and substitute $x = \cos\theta$ to obtain AG CSO.

A1

For M1 mark; cos2θ (eventually) in form $a\cos^2\theta + b$; 19sinθ sin2θ in form $c\cos\theta\sin^2\theta$ and use $\sin^2\theta = 1 - \cos^2\theta$ to obtain $c\cos\theta(1 - \cos^2\theta)$

4

(c) $16x^3 + 11x - 15 = (4x - 3)(4x^2 + 3x + 5)$

Factorise f(x)

M1A1

$b^2 - 4ac = 3^2 - 4 \times 4 \times 5 \quad (= -71)$

Find discriminant of quadratic factor; or seen in formula

m1

$b^2 - 4ac\ x^2 + 3x + 5 = 0)$

Conclusion; CSO

$\Rightarrow$ (only) solutionis $\cos\theta = \frac{3}{4}$

Condone $\frac{3}{4}$ is (only) solution

A1

M1 $(4x - 3)(4x^2 + kx \pm 5)$ A1 fully correct

m1 candidate's values of a, b, c used in expression for $b^2 - 4ac$

or complete square to obtain $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

A1 $b^2 - 4ac$ correct or $\left(x + \frac{3}{8}\right)^2 = \frac{9}{64} - \frac{5}{4} \quad \left(= -\frac{71}{64}\right)$ *and stated*

to be negative so no solution or solutions are not real (imaginary)
Accept imaginary solutions from calculator if stated to be imaginary.

Condone $\sqrt{-71}$ *is negative, or similar, so no solution.*

Conclusion x = $\frac{3}{4}$ *is solution, or cosθ =* $\frac{3}{4}$ *is solution.*

4

[10]

M12.(8sec x − 2sec^2 x = tan^2 x − 2)

8sec x − 2sec^2 x = sec^2 x − 1 − 2

Using tan^2 x = sec^2 x − 1 and NOT replacing sec^2 x with 1 + tan^2 x.

M1

3sec^2 x − 8sec x − 3(= 0)

A1

(3sec x + 1)(sec x − 3)(= 0)

Correct factors or correct use of quadratic equation formula or completing the square for 'their' equation.

sec x − $\frac{8}{6} = \pm\sqrt{\frac{64}{36} + 1}$

m1

Or sec x = $\frac{8 \pm \sqrt{(-8)^2 - 4(3)(-3)}}{2(3)}$

sec x = 3, − $\frac{1}{3}$ (or − 0.33)

Both correct.

A1

sec x = $\frac{1}{\cos x}$

PI

B1

$\left(\cos x = \frac{1}{3} \text{ or } 0.33\right)$

$\left(\sec x = -\frac{1}{3} \text{ is impossible}\right)$

x = 1.23 , 5.05

One correct. Must have earned A1 for correct quadratic, but independent of the second A1.

A1

Both correct and no extras in 0 x π.
CAO

A1

[7]

M13.(a) $\sec^2 \theta - 1 = \dots$

correct use of $\sec^2\theta = 1 + \tan^2\theta$

B1

$\sec^2\theta + 3\sec\theta - 10\ (= 0)$

quadratic expression in $\sec\theta$ *with all terms on one side*

M1

$(\sec\theta + 5)(\sec\theta - 2) = 0$

attempt at factors of their quadratic, $(\sec\theta \pm 5)(\sec\theta \pm 2)$*,*
or correct use of quadratic formula

m1

$\sec\theta = -5, 2$

A1

$\left(\cos\theta = -\frac{1}{5}, \frac{1}{2}\right)$

60°, 300°, 101.5°, 258.5° (AWRT)

3 correct, ignore answers outside interval

B1

all correct, no extras in interval

B1

6

(b) $4x - 10° = 60°, 101\cdot 5°, 258\cdot 5°, 300°$

$4x - 10$ = any of their (60),

M1

$4x$ = 70°, 111·5 °, 268· 5°, 310°

all their answers from (a), BUT must have scored B1

A1F

x = 17· 5°, 27· 9°, 67.1°, 77· 5° (AWRT)

CAO, ignore answers outside interval

A1

[9]