P3 Differentiation Revision

Name: ______________________

Class: ______________________

Date: ______________________

Time: **141 minutes**

Marks: **119 marks**

Comments:

Q1. (a) (i) Find $\frac{dy}{dx}$ when $y = xe^{2x}$.

(3)

(ii) Find an equation of the tangent to the curve $y = xe^{2x}$ at the point $(1, e^2)$

(2)

(b) Given that $y = \frac{2\sin 3x}{1 + \cos 3x}$, use the quotient rule to show that

$$\frac{dy}{dx} = \frac{k}{1 + \cos 3x}$$

where k is an integer.

(4)

(Total 9 marks)

Q2. A curve is defined by the parametric equations

$$x = t^3 + 2 \quad , \qquad y = t^2 - 1$$

(a) Find the gradient of the curve at the point where $t = -2$

(4)

Q3. A curve is defined by the equation $2y + e^{2x}y^2 = x^2 + C$, where C is a constant.

The point $P = \left(1, \frac{1}{e}\right)$ lies on the curve.

(a) Find the exact value of C.

(1)

(b) Find an expression for $\frac{dy}{dx}$ in terms of x and y.

(7)

(c) Verify that $P = \left(1, \frac{1}{e}\right)$ is a stationary point on the curve.

(2)
(Total 10 marks)

Q4. (a) Differentiate ln x with respect to x.

(1)

(b) Given that $y = (x + 1) \ln x$, find $\frac{dy}{dx}$

(2)

(c) Find an equation of the normal to the curve $y = (x + 1) \ln x$ at the point where $x = 1$.

(4)
(Total 7 marks)

Q5. A curve is defined by the equation $4x^2 + y^2 = 4 + 3xy$.

Find the gradient at the point (1, 3) on this curve.

(Total 5 marks)

Q6. A curve has equation $y = e^{-4x}(x^2 + 2x - 2)$.

(a) Show that $\frac{dy}{dx} = 2e^{-4x}(5 - 3x - 2x^2)$.

(3)

(b) Find the exact values of the coordinates of the stationary points of the curve.

(5)

(Total 8 marks)

Q7. A curve is defined by the equation

$$x^2 + xy = e^y$$

Find the gradient at the point (–1, 0) on this curve.

(Total 5 marks)

Q8. A curve has equation $y = \dfrac{2x + 3}{4x^2 + 7}$

(a) (i) Find $\dfrac{dy}{dx}$

(2)

(ii) Hence show that y is increasing when $4x^2 + 12x - 7 < 0$

(4)

(b) Find the values of x for which y is increasing.

(2)

(Total 8 marks)

Q9. A curve has equation $y = x^3 \ln x$.

(a) Find $\frac{dy}{dx}$.

(2)

(b) (i) Find an equation of the tangent to the curve $y = x^3 \ln x$ at the point on the curve where $x = e$.

(3)

(ii) This tangent intersects the x-axis at the point A. Find the exact value of the x-coordinate of the point A.

(2)

(Total 7 marks)

Q10. (a) Find $\frac{dy}{dx}$ when $y = (x^3 - 1)^6$.

(2)

(b) A curve has equation $y = x \ln x$.

(i) Find $\frac{dy}{dx}$.

(2)

(ii) Find an equation of the tangent to the curve $y = x \ln x$ at the point on the curve where $x = e$.

(3)
(Total 7 marks)

Q11.(a) Given that $x = \frac{1}{\sin\theta}$, use the quotient rule to show that $\frac{dx}{d\theta} = -\operatorname{cosec}\theta\cot\theta$.

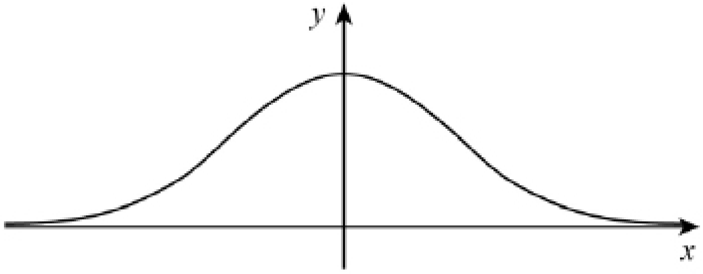
(3)

(b) Use the substitution $x = \operatorname{cosec}\theta$ to find $\int_{\sqrt{2}}^{2} \frac{1}{x^2\sqrt{x^2-1}}\,dx$, giving your answer to three significant figures.

(9)

(Total 12 marks)

Q12. The diagram shows part of the graph of $y = e^{-x^2}$

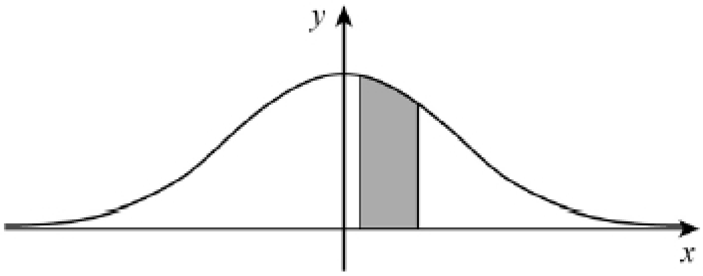


The graph is formed from two convex sections, where the gradient is increasing, and one concave section, where the gradient is decreasing.

(a) Find the values of *x* for which the graph is concave.

(4)

(b) The finite region bounded by the *x*-axis and the lines *x* = 0.1 and *x* = 0.5 is shaded.



Use the trapezium rule, with 4 strips, to find an estimate for $\int_{0.1}^{0.5} e^{-x^2}\,dx$

Give your estimate to four decimal places.

(3)

(c) Explain with reference to your answer in part **(a)**, why the answer you found in part **(b)** is an underestimate.

(2)

(d) By considering the area of a rectangle, and using your answer to part **(b)**, prove that the shaded area is 0.4 correct to 1 decimal place.

(3)
(Total 12 marks)

Q13.(a) Given that $y = x^4 \tan 2x$, find $\frac{dy}{dx}$.

(3)

(b) Find the gradient of the curve with equation $y = \frac{x^2}{x - 1}$ at the point where $x = 3$.

(3)

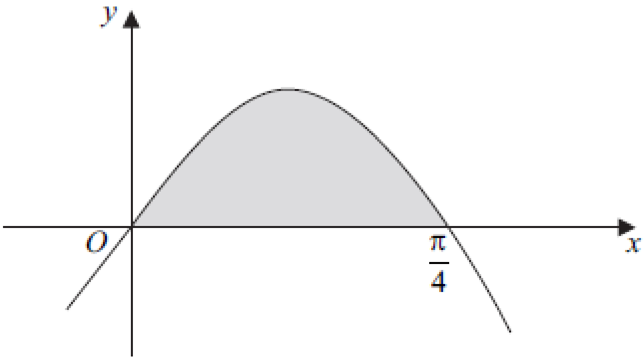
(Total 6 marks)

Q14. A curve has equation $y = 4x \cos 2x$.

(a) Find an exact equation of the tangent to the curve at the point on the curve where $x = \frac{\pi}{4}$.

(5)

(b) The region shaded on the diagram below is bounded by the curve $y = 4x \cos 2x$ and the x-axis from $x = 0$ to $x = \frac{\pi}{4}$.



By using integration by parts, find the exact value of the area of the shaded region.

(5)
(Total 10 marks)

Q15.(a) Find $\frac{dy}{dx}$ when

$y = e^{3x} + \ln x$

(2)

(b) (i) Given that $u = \frac{\sin x}{1 + \cos x}$, show that $\frac{du}{dx} = \frac{1}{1 + \cos x}$.

(3)

(ii) Hence show that if $y = \ln\left(\frac{\sin x}{1 + \cos x}\right)$, then $\frac{dy}{dx}$ = cosec x.

(2)
(Total 7 marks)

M1. (a) (i) $y = xe^{2x}$

$\left(\frac{dy}{dx} =\right) 2xe^{2x} + e^{2x}$

$kxe^{2x} + le^{2x}$ where k and l are 1s or 2s

M1

A1

$\left.\begin{matrix} k = 2 \\ l = 1 \end{matrix}\right\}$ *Independent of each other*

A1

$(= e^{2x}(2x + 1))$

ISW

3

(ii) $x = 1 \Rightarrow \frac{dy}{dx} = 3e^2$

correct substitution of $x = 1$ into their $\frac{dy}{dx}$ but must have earned M1 in part (i)

M1

tangent: $y - e^2 = 3e^2(x - 1)$ OE

CSO (no ISW), must have scored first 4 marks

common correct answer: $y = 3e^2 x - 2e^2$

A1

2

(b) $y = \frac{2\sin 3x}{1 + \cos 3x}$

$\left(\frac{dy}{dx} =\right) \frac{(1 + \cos 3x)6\cos 3x - 2\sin 3x(-3\sin 3x)}{(1 + \cos 3x)^2}$

$$\frac{\pm p(1+\cos 3x)\cos 3x \pm q\sin 3x(\sin 3x)}{(1+\cos 3x)^2}$$

where p and q are rational numbers
condone poor use/omission of brackets
PI by further working

M1

$$= \frac{6\cos 3x + 6\cos^2 3x + 6\sin^2 3x}{(1+\cos 3x)^2}$$

this line must be seen in this form (ie in terms of $\cos^2 3x$ and $\sin^2 3x$), but allow $\sin^2 3x$ replaced by $1 - \cos^2 3x$
condone denominator correctly expanded

A1

correct use of $k\sin^2 3x + k\cos^2 3x = k$
or $k\sin^2 3x = k(1 - \cos^2 3x)$

m1

$$= \frac{6\cos 3x + 6}{(1+\cos 3x)^2}$$

$$= \frac{6}{1+\cos 3x}$$

CSO

A1

4

note: if degrees used then no marks in (a) and (c)

[9]

M2.

	Marking Instructions	AO	Marks	Typical Solution
(a)	Uses a correct method for finding $\frac{dy}{dx}$ evidence for this includes sight of $\frac{dy}{dx}$ or $\frac{dx}{dt}$ and chain rule OR an attempt at implicit or explicit differentiation of a correct Cartesian equation or 'their' equation from part **(b)**	AO1.1a	M1	$\frac{dx}{dt} = 3t^2$ $\quad \frac{dy}{dt} = 2t$ $\frac{dy}{dx} = \frac{2t}{3t^2}$ When $t = -2$ $\frac{dy}{dx} = -\frac{1}{3}$ **ALT** $y = (x-2)^{\frac{2}{3}} - 1$
	Obtains correct $\frac{dy}{dx}$	AO1.1b	A1	$\frac{dy}{dx} = \frac{2(x-2)^{-\frac{1}{3}}}{3}$ When $t = -2$, $x = -6$ $\frac{dy}{dx} = \frac{2(-6-2)^{-\frac{1}{3}}}{3} = -\frac{1}{3}$
	Substitutes $t = -2$ (or $x = -6$) into 'their' equation for $\frac{dy}{dx}$	AO1.1a	M1	
	Obtains correct simplified gradient of the curve FT 'their' equation for $\frac{dy}{dx}$	AO1.1b	A1F	
(b)	Eliminates t or makes t the subject in one expression (evidence for this includes one equation with t as the subject or two equations with equal powers of t.)	AO1.1a	M1	$t^3 = (x - 2)$, $t^2 = (y + 1)$ $t^6 = (x - 2)^2$, $t^6 = (y + 1)^3$ $(x - 2)^2, = (y + 1)^3$
	Finds a correct Cartesian equation in any form	AO1.1b	A1	**ALT** $t^3 = (x - 2)$ $t = (x-2)^{\frac{1}{3}}$ $y = (x-2)^{\frac{2}{3}} - 1$
				Total 6 marks

M3. (a) $(C=)\dfrac{2}{e}$ or $2e^{-1}$ or $2\left(\dfrac{1}{e}\right)$ or $2\left(e^{-1}\right)$

One of these answers only.
Not 0.736 but allow ISW.

B1

1

(b) $\dfrac{d}{dx}(2y) = 2\dfrac{dy}{dx}$

B1

$\dfrac{d}{dx}\left(e^{2x}y^2\right) = 2e^{2x}y^2 + e^{2x}2y\dfrac{dy}{dx}$

Product; 2 terms added, one with $\dfrac{dy}{dx}$ *;*

M1

A1 for each term

A1

A1

$\dfrac{d}{dx}\left(x^2 + C\right) = 2x$

B1

$\dfrac{dy}{dx} =$

Solve their equation correctly for $\dfrac{dy}{dx}$

M1

$\dfrac{x - e^{2x}y^2}{e^{2x}y + 1}$

Condone factor of 2 in both numerator and denominator. ISW

A1

7

(c) Evaluate $\frac{dy}{dx}$ at $\left(1, \frac{1}{e}\right)$

Substitute $x = 1$ *and* $y = \frac{1}{e}$ *into numerator of* $\frac{dy}{dx}$*;*
allow one slip

M1

numerator $= 1 - e^2 e^{-2} = 0 \Rightarrow$ stationary point

Conclusion required; must score full marks in part (b)
Allow 1 – 1 = 0 or 2 – 2 = 0

A1

2

[10]

M4. (a) $y = \ln x$

penalise + c once on 1 (a) or 2 (a)

$\frac{dy}{dx} = \frac{1}{x}$

B1

1

(b) $y = (x + 1) \ln x$

$\frac{dy}{dx} = (x+1) \times \frac{1}{x} + \ln x$

product rule

M1A1

2

(c) $y = (x + 1)\ln x$

$\frac{dy}{dx} = \frac{1}{x} + 1 + \ln x$

$x = 1$: $\frac{dy}{dx} = 1 + 1 = 2$

substitute $x = 1$ into their $\frac{dy}{dx}$

M1

Grad normal = $-\frac{1}{2}$

use of $m_1m_2 = -1$

M1

CSO

A1

$y = -\frac{1}{2}(x - 1)$

OE

A1

4

[7]

M5. $8x + 2y\frac{dy}{dx} = 3y + 3x\frac{dy}{dx}$

$8x$ and $4 \to 0$

B1

$2y\frac{dy}{dx}$

B1

$3y + 3x\frac{dy}{dx}$

Two terms with one $\frac{dy}{dx}$

M1A1

at (1, 3) (gradient) $\frac{dy}{dx} = \frac{1}{3}$

CSO

A1

[5]

M6. (a) $y' = e^{-4x}(2x + 2) - 4e^{-4x}(x^2 + 2x - 2)$

$y' = Ae^{-4x}(ax + b) \pm Be^{-4x}(x^2 + 2x - 2)$
where A and B are non-zero constants

M1

All correct

A1

$= e^{-4x}(2x + 2 - 4x^2 - 8x + 8)$

or $-4x^2e^{-4x} - 6xe^{-4x} + 10e^{-4x}$

$= 2e^{-4x}(5 - 3x - 2x^2)$

AG; all correct with no errors,
2nd line (OE) must be seen
Condone incorrect order on final line

A1

or

$y = x^2 e^{-4x} + 2xe^{-4x} - 2e^{-4x}$

$y' = -4x^2e^{-4x} + 2xe^{-4x} + 2x. - 4e^{-4x} + 2e^{-4x} + 8e^{-4x}$

$Ax^2e^{-4x} + Bxe^{-4x} + Cxe^{-4x} + De^{-4x} + Ee^{-4x}$

(M1)

All correct

(A1)

$= -4x^2 e^{-4x} - 6xe^{-4x} + 10e^{-4x}$

$=2e^{-4x}(5 - 3x - 2x^2)$

AG; all correct with no errors,
3rd line (OE) must be seen

(A1)

3

(b) $-(2x + 5)(x - 1)$ (= 0)

OE Attempt at factorisation
(±2x ± 5)(±x ± 1)
or formula with at most one error

M1

$x = \frac{-5}{2}, 1$

Both correct and no errors
SC x = 1 only scores M1A0

A1

$x = 1, y = e^{-4}$

For $y = ae^b$ attempted

m1

Either correct, follow through only from incorrect sign for x

A1F

$x = -\frac{5}{2},\ y = e^{10}\left(-\frac{3}{4}\right)$

CSO 2 solutions only
Note: withhold final mark for extra solutions
Note: approximate values only for y can score m1 only

A1

5

[8]

M7. $x^2 + xy = e^y$

$2x + y + x\frac{dy}{dx} = e^y \frac{dy}{dx}$

2x

B1

Use product rule

M1A1

RHS

B1

$(-1, 0)$ $\frac{dy}{dx} = -1$

CSO

A1

[5]

M8.

	Marking Instructions	AO	Marks	Typical Solution
(a) (i)	Selects an appropriate routine procedure; evidence of quotient rule or product rule	AO1.1a	M1	$\frac{dy}{dx} = \frac{2(4x^2+7)-8x(2x+3)}{(4x^2+7)^2}$
	Obtains correct derivative (no need for simplification)	AO1.1b	A1	
(ii)	States clearly that $\frac{dy}{dx} > 0 \Rightarrow y$ is increasing	AO2.4	R1	y is increasing $\Leftrightarrow \frac{dy}{dx} > 0$ $\frac{2(4x^2+7)-8x(2x+3)}{(4x^2+7)^2} > 0$
	Forms inequality from 'their'	AO3.1a	B1	$(4x^2 + 7)^2 > 0$ for all x

	$\frac{dy}{dx} > 0$			$\therefore 2(4x^2 + 7) - 8x(2x + 3) > 0$ $8x^2 + 14 - 16x^2 - 24x > 0$ $4x^2 + 12x - 7 < 0$ **(AG)**
	Deduces numerator must be positive	AO2.2a	R1	
	Considers denominator alone and sets out clear argument to justify given inequality **AG** Only award this mark if they have a completely correct solution, which is clear, easy to follow and contains no slips	AO2.1	R1	
(b)	Solves the correct quadratic inequality (accept evidence of factorising, completing the square, use of formula, or correct critical values stated)	AO1.1a	M1	$(2x + 7)(2x - 1)$ $x = -\frac{7}{2}, \frac{1}{2}$
	Obtains fully correct answer, given as an inequality or using set notation	AO1.1b	A1	$-\frac{7}{2} < x < \frac{1}{2}$ **Or** $x \in \left(-\frac{7}{2}, \frac{1}{2}\right)$ **Or** $x \in \left\{x : -\frac{7}{2} < x < \frac{1}{2}\right\}$
				Total 8 marks

M9.(a) $\left(\frac{dy}{dx} =\right) x^3 \times \frac{1}{x} + 3x^2 \ln x$

$px^3 \times \frac{1}{x} + qx^2 \ln x$

where p and q are integers

M1

$p = 1, q = 3$

A1

2

(b) (i) $\left(\frac{dy}{dx} =\right)$ $e^2 + 3e^2 \ln e$ $(= 4e^2)$

Substituting e for x in their $\frac{dy}{dx}$*, but must have scored M1 in (a)*

M1

$y = e^3 \ln e$ $(= e^3)$

B1

$y - e^3 = 4e^2 (x - e)$

OE ***but*** *must have evaluated ln e (twice) for this mark (must be in exact form, but condone numerical evaluation after correct equation)*

A1 **3**

(ii) $- e^3 = 4e^2 (x - e)$ or $4e^2 x = 3e^3$ OE

Correctly substituting y = 0 into a correct tangent equation in (b)(i)

M1

$x = \frac{3}{4}$ e

CSO;

ignore subsequent decimal evaluation

A1 **2**

[7]

M10. (a) $\frac{dy}{dx} = k(x^3 - 1)^5$

Where k is an integer or function of x

M1

$= 6 \times 3x^2(x^3 - 1)^5$ (ISW)

But note

$\frac{dy}{dx} = k(x^3 - 1)^5 + px^2$ *M0*

Or

(u = x^3 – 1) *(y = u^6)*

$\frac{dy}{du} = 6u^5$ *and* $\frac{du}{dx} = 3x^2$ *M1*

= 6(x^3 – 1)^5 × 3x^2 *A1*

Note

$\frac{dy}{dx} = 6 \times 3x^2 (x^3 - 1)^5 + c$ *scores* *M1 A0*

(penalise + c in differential once only in paper)

A1

2

(b) (i) $\frac{dy}{dx} = \pm x \times \frac{1}{x} \pm \ln x$

*Product rule attempted **and** differential of ln x*

M1

= 1 + ln x (ISW)

A1

2

(ii) (x = e) y = e PI

Must have replaced ln e by 1

Condone y = 2.72 (AWRT)

B1

$\frac{dy}{dx} = 1 + \ln e (= 2)$

Correct substitution into their $\frac{dy}{dx}$

But must have scored M1 in (b)(i)

M1

y – e = 2(x – e) or y = 2x – e OE, ISW

Must have replaced ln e by 1

A1

3

[7]

M11.(a) $\left(\frac{dx}{d\theta}=\right)\frac{(\sin\theta\times 0)-1\times\cos\theta}{\sin^2\theta}$

quotient rule $\frac{\pm\sin\theta\times k\pm 1\times\cos\theta}{\sin^2\theta}$ *where* k *= 0 or 1*

M1

must see the '0' either in the quotient or in eg $\frac{du}{d\theta}$ *= 0 etc*

A1

$=-\frac{\cos\theta}{\sin^2\theta}$ or $=-\frac{\cos\theta}{\sin\theta\sin\theta}$

or equivalent

$= -\operatorname{cosec}\theta\cot\theta$

CSO, AG must see one of the previous expressions

A1

3

(b) $x = \operatorname{cosec}\theta$

$\frac{dx}{d\theta} = -\operatorname{cosec}\theta\cot\theta$

OE, eg $dx = -\operatorname{cosec}\theta\cot\theta\, d\theta$

B1

Replacing $\sqrt{(\text{cosec}^2\theta - 1)}$ by $\sqrt{\cot^2\theta}$, or better

at any stage of solution

B1

$\int = \int \frac{-\text{cosec}\,\theta \cot\theta}{\text{cosec}^2\theta\sqrt{(\text{cosec}^2\theta - 1)}}\,\text{d}\theta$

all in terms of θ, and including their attempt at dx, but condone omission of dθ

M1

fully correct and must include dθ (at some stage in solution)

A1

$\int \frac{-\text{cosec}\,\theta \cot\theta}{\text{cosec}^2\theta \cot\theta}(\text{d}\theta) = \int \frac{-1}{\text{cosec}\,\theta}\ (\text{d}\theta)$

OE eg $\int -\sin\theta\ (\text{d}\theta)$

A1

= $\cos\theta$

A1

$x = 2$, $\theta = 0.524$ AWRT

$x = \sqrt{2}$, $\theta = 0.785$ AWRT

correct change of limits or (±)cosθ = $(\pm)\left[\sqrt{\left(1 - \frac{1}{x^2}\right)}\right]_{\sqrt{2}}^{2}$ *OE*

B1

0.8660 − 0.7071

c's F(0.52) − F(0.79)

substitution into ± cosθ only or $\left(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right)$

m1

= 0.159

A1

9

[12]

M12.

	Marking Instructions	AO	Marks	Typical Solution
(a)	Finds 2nd derivative and sets up an inequality	AO3.1a	M1	$\frac{dy}{dx} = -2xe^{-x^2}$
	Obtains correct first derivative	AO1.1b	A1	$\frac{d^2y}{dx^2} = -2e^{-x^2} + 4x^2e^{-x^2}$
	Obtains second derivative correct from their first derivative	AO1.1b	A1F	$-2e^{-x^2} + 4x^2e^{-x^2} < 0$ $4x^2 - 2 < 0$
	Deduces correct final inequality (could use set notation)	AO2.2a	A1	$-\frac{\sqrt{2}}{2} < x < \frac{\sqrt{2}}{2}$
(b)	Uses trapezium rule	AO1.1a	M1	$\int_{0.1}^{0.5} e^{-x^2}dx \approx \frac{0.1}{2}(e^{-0.01} + e^{-0.25}$
	Trapezium rule entries all correct	AO1.1b	A1	$+2(e^{-0.04} + e^{-0.09} + e^{-0.16}))$ ≈ 0.3616
	Finds correct value	AO1.1b	A1	
(c)	References area being completely within concave section So...	AO2.4	E1	$[0.1, 0.5] \subset \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$ ∴ area is completely within concave section
	Trapezia all fall completely underneath the curve therefore underestimate (only award this mark if previous E1 has been awarded)	AO2.4	E1	Hence trapezia lie below curve and give an under-estimate for the area
(d)	Uses suitable rectangle to obtain over-estimate	AO3.1a	B1	Using a rectangle with the left hand edge the same

Explains that this rectangle lies above the curve	AO2.4	E1	height as the curve will produce an over-estimate Area of rectangle = $0.4 \times e^{-0.01^2} = 0.396...$ $\therefore 0.36 < A < 0.40$ So A = 0.4 to 1 d.p
Constructs rigorous mathematical argument about accuracy, which leads to required result Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips.	AO2.1	R1	
Total 12 marks			

M13.(a) ($y = x^4 \tan 2x$)

$\left(\frac{dy}{dx} =\right)$ $4x^3 \tan 2x + x^4\, 2\sec^2 2x$

$4x^3 \tan 2x + Ax^4 \sec^2 kx$
OE where A is a non-zero constant.

M1

A1 for $k = 2$
may have $(\sec 2x)^2$
or $\frac{1}{\cos^2 2x}$

A1

A1 all correct
ISW if attempt to simplify is incorrect.

A1

3

(b) $\left(\frac{dy}{dx} =\right)\frac{\pm 2x(x-1) \pm 1(x^2)}{(x-1)^2}$

Use of the quotient rule

M1

$$\frac{2x(x-1)-1(x^2)}{(x-1)^2}$$

A1

$$\left(=\frac{x^2-2x}{(x-1)^2}\right)$$

Simplification not required

$\left(\frac{dy}{dx}=\right)\frac{3}{4}$ or 0.75 OE

Obtained from correct $\frac{dy}{dx}$

A1

3

[6]

M14.(a) $y = 4x\cos 2x$

$\left(\frac{dy}{dx}=\right)$ $4\cos 2x - 4x(2)\sin 2x$

anything reducible to $A\cos 2x + Bx \sin 2x$ *where* A *and* B *are non-zero integers*

M1

gradient of the tangent

OE, all correct

A1

$A\cos\frac{2\pi}{4}+B\times\frac{\pi}{4}\sin\frac{2\pi}{4}$

substituting $\frac{\pi}{4}$ *into candidate's derived function*

m1

$= -2\pi$

must have $= -2\pi$ *using correct* $\frac{dy}{dx}$

A1

an equation of the tangent is $y = -2\pi\left(x - \frac{\pi}{4}\right)$

OE, dependent on previous A1

A1

5

(b)

$\left(\int_0^{\frac{\pi}{4}} 4x\cos 2x \, dx\right)$

$u = Ax$ $\quad \frac{dv}{dx} = \cos 2x$

$\frac{du}{dx} = A$ $\quad v = B\sin 2x$

all 4 terms in this form seen or used

M1

A *= 4 and* $B = \frac{1}{2}$ ***or*** A *= 1 and* B *= 2, etc*

A1

$$= \left[4x\frac{1}{2}\sin 2x\right]_{(0)}^{\left(\frac{\pi}{4}\right)} - \int_{(0)}^{\left(\frac{\pi}{4}\right)} 4 \times \frac{1}{2}\sin 2x(\mathrm{d}x)$$

correct substitution of candidate's terms into integration by parts formula condone missing limits

m1

$$= \left[4x\frac{1}{2}\sin 2x\right]_{(0)}^{\left(\frac{\pi}{4}\right)} - \left[-\cos 2x\right]_{(0)}^{\left(\frac{\pi}{4}\right)}$$

candidate's second integration completed correctly
FT on one error including coefficient condone missing limits

A1F

$$= \frac{\pi}{2} - 1$$

OE, exact value

A1

5

[10]

M15.(a) $\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right) 3e^{3x} + \frac{1}{x}$

B1 for one term correct

B1

B1 all correct

B1

2

(b) (i) $\left(\frac{\mathrm{d}u}{\mathrm{d}x} =\right) \frac{\pm\cos x(1+\cos x) \pm \sin x(\sin x)}{(1+\cos x)^2}$

clear attempt at quotient / product rule *condone poor use of brackets*

M1

$$\frac{\cos x(1+\cos x)-\sin x(-\sin x)}{(1+\cos x)^2}$$

any correct form seen

A1

$$= \frac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2} = \frac{\cos x+1}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

AG be convinced
correct use of brackets and correct notation used throughout (eg A0 if cos x^2 etc seen)

A1cso

3

(ii) $\left(\frac{dy}{dx}=\right)\frac{1+\cos x}{\sin x}\times\frac{1}{1+\cos x}$ OE

correct use of chain rule

M1

$= \frac{1}{\sin x}$

= cosec x

AG, must see = $\frac{1}{\sin x}$ *and no errors seen;*
condone incorrect use of brackets only if penalised in part (b)(i)

A1

2

[7]