P3 Integration Revision Questions

Name: ______________________

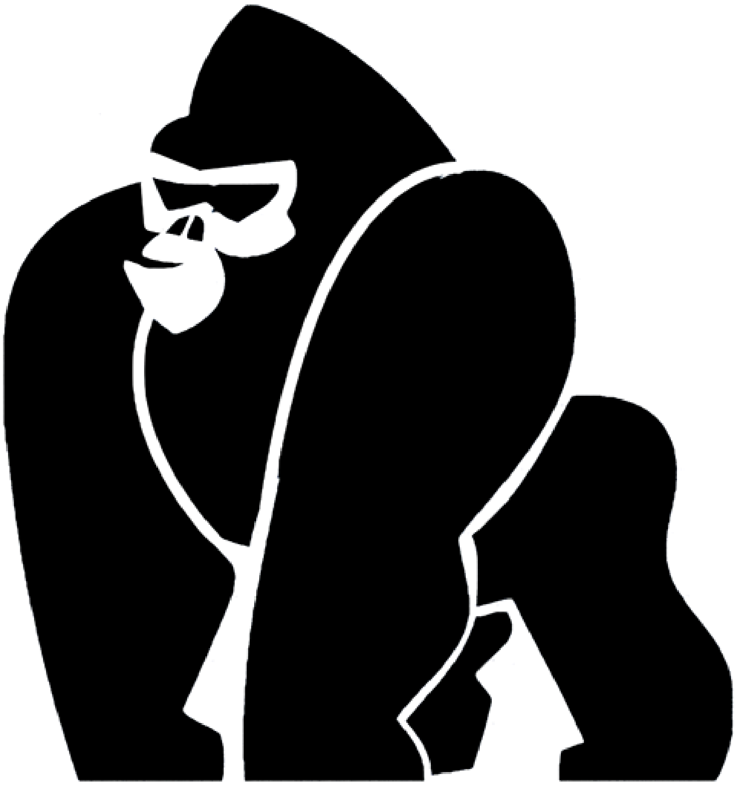
Class: ______________________

Date: ______________________

Time: **178 minutes**

Marks: **150 marks**

Comments:



Q1.(a) Given that $u = 2^x$, write down an expression for $\frac{du}{dx}$

(1)

(b) Find the exact value of $\int_0^1 2^x\sqrt{3+2^x}\,dx$

Fully justify your answer.

(6)
(Total 7 marks)

Q2. (a) Express $\frac{1}{(3-2x)(1-x)^2}$ in the form $\frac{A}{3-2x}+\frac{B}{1-x}+\frac{C}{(1-x)^2}$.

(4)

(b) Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\sqrt{y}}{(3-2x)(1-x)^2}$$

where $y = 0$ when $x = 0$, expressing your answer in the form

$$y^p = q\ln[\mathrm{f}(x)]+\frac{x}{1-x}$$

where *p* and *q* are constants.

(9)

(Total 13 marks)

Q3. (a) Using integration by parts, find $\int x \sin(2x-1)\, dx$.

(5)

(b) Use the substitution $u = 2x - 1$ to find $\int \frac{x^2}{2x-1}\, dx$, giving your answer in terms of x.

(6)

(Total 11 marks)

Q4. Use the substitution $u = 1 + 2 \tan x$ to find

$$\int \frac{1}{(1+2\tan x)^2 \cos^2 x} \mathrm{d}x$$

(Total 5 marks)

Q5. Solve the differential equation

$$\frac{dy}{dx} = y^2 x\sin 3x$$

given that $y = 1$ when $x = \frac{\pi}{6}$. Give your answer in the form $y = \frac{9}{f(x)}$.

(Total 9 marks)

Q6. (a) Find $\int \frac{1}{3+2x} \mathrm{d}x$.

(2)

(b) By using integration by parts, find $\int x \sin \frac{x}{2} \mathrm{d}x$.

(4)

(Total 6 marks)

Q7.(a) (i) Express $\frac{5x-6}{x(x-3)}$ in the form $\frac{A}{x}+\frac{B}{x-3}$.

(2)

(ii) Find $\int \frac{5x-6}{x(x-3)}\,\mathrm{d}x$.

(2)

(b) (i) Given that

$$4x^3 + 5x - 2 = (2x + 1)(2x^2 + px + q) + r$$

find the values of the constants p, q and r.

(4)

(ii) Find $\int \frac{4x^3+5x-2}{2x+1}\,\mathrm{d}x$.

(3)

(Total 11 marks)

Q8. Use the substitution $u = x^4 + 2$ to find the value of $\int_0^1 \frac{x^7}{(x^4+2)^2}$ dx, giving your answer in the form $p \ln q + r$, where p, q and r are rational numbers.

(Total 6 marks)

Q9.(a) Express $\frac{2x+3}{4x^2-1}$ in the form $\frac{A}{2x-1}+\frac{B}{2x+1}$, where A and B are integers.

(3)

(b) Express $\frac{12x^3-7x-6}{4x^2-1}$ in the form $Cx+\frac{D(2x+3)}{4x^2-1}$, where C and D are integers.

(3)

(c) Evaluate $\int_1^2 \frac{12x^3-7x-6}{4x^2-1}\,dx$, giving your answer in the form $p + \ln q$, where p and q are rational numbers.

(5)

(Total 11 marks)

Q10.(a) (i) Express $\frac{5-8x}{(2+x)(1-3x)}$ in the form $\frac{A}{2+x}+\frac{B}{1-3x}$, where A and B are integers.

(3)

(ii) Hence show that $\int_{-1}^{0}\frac{5-8x}{(2+x)(1-3x)}\,dx = p\ln 2$, where p is rational.

(4)

(b) (i) Given that $\frac{9-18x-6x^2}{2-5x-3x^2}$ can be written as $C+\frac{5-8x}{2-5x-3x^2}$, find the value of C.

(1)

(ii) Hence find the exact value of the area of the region bounded by the curve $y=\frac{9-18x-6x^2}{2-5x-3x^2}$, the x-axis and the lines $x = -1$ and $x = 0$.

You may assume that $y > 0$ when $-1 \le x \le 0$.

(2)
(Total 10 marks)

Q11.(a) Given that $x = \frac{1}{\sin\theta}$, use the quotient rule to show that $\frac{\mathrm{d}x}{\mathrm{d}\theta} = -\operatorname{cosec}\theta\cot\theta$.

(3)

(b) Use the substitution $x = \operatorname{cosec}\theta$ to find $\int_{\sqrt{2}}^{2} \frac{1}{x^2\sqrt{x^2-1}}\,\mathrm{d}x$, giving your answer to three significant figures.

(9)

(Total 12 marks)

Q12.(a) Given that $y = 4x^3 - 6x + 1$, find $\frac{dy}{dx}$.

(1)

(b) Hence find $\int_2^3 \frac{2x^2 - 1}{4x^3 - 6x + 1} dx$, giving your answer in the form $p \ln q$,

where p and q are rational numbers.

(5)
(Total 6 marks)

Q13.(a) Show that

$$\int_0^{\ln 2} e^{1-2x}\,dx = \frac{3}{8}e$$

(4)

(b) Use the substitution *u* = tan *x* to find the exact value of

$$\int_0^{\frac{\pi}{4}} \sec^4 x\sqrt{\tan x}\,dx$$

(8)
(Total 12 marks)

Q14.(a) Find $\int x\sqrt{x^2+3}\,\mathrm{d}x$.

(2)

(b) Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x\sqrt{x^2+3}}{\mathrm{e}^{2y}}$$

given that $y = 0$ when $x = 1$. Give your answer in the form $y = \mathrm{f}(x)$.

(7)
(Total 9 marks)

Q15. The height $\frac{dx}{dt} = \frac{8 \sin 2t}{3\sqrt{x}}$f a column of water in a fountain display satisfies the differential equation , where *t* is the time in seconds after the display begins.

(a) Solve the differential equation, given that initially the column of water has zero height.
Express your answer in the form $x = \mathrm{f}(t)$

(7)

(b) Find the maximum height of the column of water, giving your answer to the nearest cm.

(1)
(Total 8 marks)

Q16.(a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_1^5 \frac{1}{x^2+1}\,\mathrm{d}x$$

giving your answer to three significant figures.

(4)

(b) (i) Find $\int \left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}} \right) \mathrm{d}x$, giving the coefficient of each term in its simplest form.

(3)

(ii) Hence find the value of $\int_1^4 \left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}} \right) \mathrm{d}x$.

(2)

(Total 9 marks)

Q17. $\int_1^2 x^3 \ln(2x)\mathrm{d}x$ can be written in the form $p\ln 2 + q$, where p and q are rational numbers.

Find p and q.

(Total 5 marks)

M1.

	Marking Instructions	**AO**	**Marks**	**Typical Solution**
(a)	States the correct derivative	AO1.1b	B1	$2^x \ln 2$
(b)	Selects an appropriate method for integrating, which could lead to a correct exact solution (this could be indicated by an attempt at a substitution or attempting to write the integrand in the form $f'(x)f(x)^n$)	AO3.1a	M1	Let $u = 2^x$ Then $\frac{du}{dx} = 2^x \ln 2$ And $\frac{1}{\ln 2}\frac{du}{dx} = 2^x$ $I = \int (3+u)^{\frac{1}{2}} \frac{1}{\ln 2}\frac{du}{dx}\,dx$ $= \frac{1}{\ln 2}\int (3+u)\frac{1}{2}\,du$ $= \frac{3}{3\ln 2}(3+u)^{\frac{3}{2}} + c$ Sub limits: $\left[\frac{2}{3\ln 2}(3+u)^{\frac{3}{2}}\right]_1^2$ $\frac{3}{2} \times \frac{1}{\ln 2}(5\sqrt{5} - 8)$ **ALT (direct inspection)** $\int 2x\sqrt{3+2^x}\,dx$ $= \frac{1}{\ln 2}\int 2^x \ln 2\sqrt{3+2^x}\,dx$ $= \frac{1}{\ln 2}\int 2^x \ln 2(3+2^x)^{\frac{1}{2}}\,dx$ $= \frac{1}{\ln 2} \times \frac{2}{3}(3+2^x)^{\frac{3}{2}}$ $\left[\frac{1}{\ln 2} \times \frac{2}{3}(3+2^x)^{\frac{3}{2}}\right]_0^1$ $\frac{3}{2} \times \frac{1}{\ln 2}(5\sqrt{5} - 8)$
	Correctly writes integrand in a form which can be integrated (condone missing or incorrect limits)	AO1.1b	A1	
	Integrates ‘their’ expression (allow one error)	AO1.1a	M1	
	Substitutes correct limits corresponding to ‘their’ method	AO1.1a	M1	
	Obtains correct value in an exact form	AO1.1b	A1	
	Mark awarded if they have a completely correct solution, which is clear, easy to follow and contains no slips Substitution should be clearly stated in exact form and change of variable or solution by direct inspection should be achieved correctly with correct use of symbols and connecting language	AO2.1	R1	
				Total 7 marks

M2. (a) $1 = A(1 - x)^2 + B(1 - x)(3 - 2x) + C(3 - 2x)$

Attempt to clear fractions

M1

$$\left.\begin{array}{lll} x = 1 & x = \frac{3}{2} & x = 0 \\ C = 1 & 1 = A\left(-\frac{1}{2}\right)^2 & 1 = A + 3B + 3C \end{array}\right\}$$

Use any two (or three) values of x to set up two (or three) equations

m1

$A = 4$ $B = -2$ $C = 1$

Two values correct

A1

All values correct

A1

Alternative

$1 = A(1 - x)^2 + B(1 - x)(3 - 2x) + C(3 - 2x)$

(M1)

$1 = A + 3B + 3C$

$0 = -2A - 5B - 2C$

Set up three simultaneous equations

(m1)

$0 = A + 2B$

Two values correct

(A1)

$A = 4 \quad B = -2 \quad C = 1$

All values correct

(A1)

4

(b) $\int \frac{1}{2\sqrt{y}}\,\mathrm{d}y = \int \frac{4}{3-2x} - \frac{2}{1-x} + \frac{1}{(1-x)^2}\,\mathrm{d}x$

Separate using partial fractions; correct notation; condone missing integral signs but dy and dx must be in correct place.

ft on their A, B, C and on each integral.

B1ft

$\int \frac{1}{2\sqrt{y}}\,\mathrm{d}y = \sqrt{y} =$

OE $\int \frac{k}{\sqrt{y}}\,\mathrm{d}y = 2k\sqrt{y}$ *is B1*

B1

$-2\ln(3 - 2x)$

Condone missing brackets on one ln integral.

B1ft

$+ 2\ln(1 - x)$

B1ft

$+ \frac{1}{1-x}(+C)$

Condone omission of +C

B1ft

$x = 0 \quad y = 0 \Rightarrow 0 = -2\ln 3 + 0 + 1 + C$

Use (0,0) to find C. Must get to C = …..

M1

$C = 2 \ln 3 - 1$

Correct C found from correct equation.
C must be exact, in any form but not decimal.

A1

$$\sqrt{y} = 2\ln\left(\frac{3-3x}{3-2x}\right) + \frac{1}{1-x} - 1$$

Correct use of rules of logs to progress towards requested form of answer. C must be of the form r lns + t

m1

$$y^{\frac{1}{2}} = 2\ln\left(\frac{3-3x}{3-2x}\right) + \frac{x}{1-x}$$

OE
CSO condone B0 for separation

A1

9

[13]

M3. (a) $\int x\sin(2x-1)\,\mathrm{d}x$

$u = x \qquad \frac{\mathrm{d}v}{\mathrm{d}x} = \sin(2x-1)$

$\int \sin f(x), \frac{d}{dx}(x)$ *attempted*

M1

$\frac{\mathrm{d}u}{\mathrm{d}x} = 1 \quad v = -\frac{1}{2}\cos(2x-1)$

All correct – condone omission of brackets

A1

$$\left(\int =\right) -\frac{x}{2}\cos(2x-1) - \int -\frac{1}{2}\cos(2x-1)\,(\mathrm{d}x)$$

correct substitution of their terms into parts

m1

$$= -\frac{x}{2}\cos(2x-1) + \frac{1}{2}\int \cos(2x-1)\,(\mathrm{d}x)$$

All correct – condone omission of brackets

A1

$$= -\frac{x}{2}\cos(2x-1) + \frac{1}{4}\sin(2x-1) + c$$

CSO condone missing + c *and dx*
Condone missing brackets around 2x – 1 if recovered in final line ISW

A1

5

(b) $u = 2x - 1$
'du = 2 dx'

OE

M1

$$\int \frac{x^2}{2x-1}\,\mathrm{d}x = \int \frac{(u+1)^2}{4u}\,\frac{\mathrm{d}u}{2}$$

All in terms of u

m1

All correct

A1

PI from later working

$$= \left(\frac{1}{8}\right)\int \frac{u^2 + 2u + 1}{u}\,\mathrm{d}u$$

$$= \left(\frac{1}{8}\right)\int u + 2 + \frac{1}{2}\,\mathrm{d}u$$

A1

$$= \left(\frac{1}{8}\right)\left[\frac{u^2}{2} + 2u + \ln u\right]$$

$$or \left(\frac{1}{8}\right)\left[\frac{(u+2)^2}{2} + \ln u\right]$$

B1

$$= \frac{1}{8}\left[\frac{(2x-1)^2}{2} + 2(2x-1) + \ln(2x-1)\right] + c$$

$$or = \frac{1}{8}\left[\frac{(2x+1)^2}{2} + \ln(2x-1)\right] + c$$

CSO condone missing + c only ISW

A1

6

[11]

M4. $$\int \frac{1}{\cos^2 x(1+2\tan x)^2}\,dx$$

$u = 1 + 2\tan x$

$$\left(\frac{du}{dx} = \right) 2\sec^2 x \text{ OE}$$

condone $\left(\frac{du}{dx} = \right) a\sec^2 x$ *where a is a constant*

M1

$$\int = \int \frac{du}{2u^2}$$

$\int \frac{k}{u^2}\,(du)$*, where k is a constant*

m1

correct, or $\frac{1}{2}\int u^{-2}\,(du)$

A1

$= \frac{1}{2}\frac{u^{-1}}{-1}$

correct integral of their expression but must have scored M1 m1

A1F

$= -\frac{1}{2u}$

$= -\frac{1}{2(1+2\tan x)}(+\,c)$

CSO, no ISW

A1

[5]

M5. $\int \frac{dy}{y^2} = \int x \sin 3x \, dx$

Correct separation and notation; condone missing integral signs

B1

$\int \frac{dy}{y^2} = -\frac{1}{y}$

B1

$\int x \sin 3x \, dx = x\left(-\frac{1}{3}\cos 3x\right)$

$u = x \quad \frac{dv}{dx} = \sin 3x$

Use parts $\frac{du}{dx} = 1 \quad v = k\cos 3x$ *with correct substitution into formula*

$-\int -\frac{1}{3}\cos 3$

A1

$= -\frac{1}{3}x\cos 3x$ dx + $\frac{1}{9}$ $\sin 3x$

CAO

A1

$-\frac{1}{y} = -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C$

$-1 = -\frac{1}{3} \times \frac{\pi}{6}\cos\left(\frac{\pi}{2}\right) + \frac{1}{9}\sin\left(\frac{\pi}{2}\right) + C$

Use $x = \frac{\pi}{6}$ $y = 1$ *to find* C

M1

$C = -\frac{10}{9}$

CAO

A1

$-\frac{1}{y} = -\frac{1}{9}$ (3$x$cos3x dx − sin3x + 10)

And invert to $-y = -\frac{9}{(\cdots\cdots)}$

m1

$y = \frac{9}{3x\cos 3x - \sin 3x + 10}$

CSO, condone first B1 not given

A1

Second M1 finding C; substitute x = $\frac{\pi}{6}$ *y = 1 into f(y) = px cos3x + qsin3x + C and evaluate using radians.*

Must calculate a value of C.

m1 for reaching form ± $\frac{k}{y} = \frac{1}{9}$*(Pxcos3x Qsin3x + R) where P and Q are ±3 or ±*$\frac{1}{3}$ *or ±1 and inverting to* $\pm\frac{y}{k} = \frac{9}{(Px\cos 3x + Q\sin 3x + R)}$

[9]

M6. (a) (i) $\int \frac{1}{3+2x}\,\mathrm{d}x$

= k ln (3 + 2x)

Where k is a rational number

M1

$= \frac{1}{2}\ln(3 + 2x) + c$

Or

if substitution u = 3 + 2x, du = 2dx

$\int = \int \frac{1}{u}\frac{du}{2} = k\ln u$ *M1*

$= \frac{1}{2}\ln(3 + 2x) + c$ *A1*

A1

2

(b) $u = x \quad \mathrm{d}v = \sin\frac{x}{2}$

$$\int sin\frac{x}{2}(dx) = k cos\frac{x}{2}, \frac{d}{dx}(x) = 1$$

where k is a constant

M1

$$du = 1 \quad v = -2\cos\frac{x}{2}$$

All correct

A1

$$\int = -2x\cos\frac{x}{2} - \int -2\cos\frac{x}{2}(dx)$$

***Correct** substitution of their terms into parts formula (watch signs carefully)*

m1

$$= -2x\cos\frac{x}{2} + 4\sin\frac{x}{2} + c$$

CAO

A1

4

[6]

M7.(a) (i) $5x - 6 = A(x - 3) + Bx$

Multiply by denominator and use two values of x.

M1

$x = 0 \quad x = 3$

$A = 2 \quad B = 3$

A1

Alternative: equate coefficients

$-6 = -3A \quad 5 = A + B$

Set up and solve simultaneous equations for values of A and B.

(M1)

$A = 2 \quad B = 3$

(A1)

2

(ii) $\left(\int \frac{2}{x} + \frac{3}{x-3}\,dx = \right) 2\ln x$

their A ln x

B1ft

$+ 3\ln(x - 3)(+C)$

their B ln $(x - 3)$ and no other terms; condone B ln $x - 3$

B1ft

2

(b) (i)

$$\begin{array}{rl} & \underline{2x^2 - x + 3} \\ 2x+1 \big) & 4x^3 + 5x - 2 \\ & \underline{4x^3 + 2x^2} \\ & \quad -2x^2 + 5x \\ & \quad \underline{-2x^2 - x} \\ & \qquad 6x - 2 \\ & \qquad \underline{6x + 3} \\ & \qquad\quad -5 \end{array}$$

Division as far as $2x^2 + px + q$ with $p \neq 0$, $q \neq 0$, PI

M1

$p = -1$

PI by $2x^2 - x + q$ seen

A1

$q = 3$

PI by $2x^2 - x + 3$ *seen*

A1

$r = -5$

and must state $p = -1$, $q = 3$, $r = -5$ *explicitly or write out full correct RHS expression*

A1

Alternative 1:

$4x^3 + 5x - 2 = 4x^3 + (2 + 2p)x^2 + (p + 2q)x + q + r$

$2 + 2p = 0$

Clear attempt to equate coefficients, PI by $p = -1$

(M1)

$p + 2q = 5$

$q + r = -2$

$p = -1$

(A1)

$q = 3 \quad r = -5$

(A1A1)

Alternative 2:

$4x^3 + 5x - 2 = (2x + 1)(2x^2 + px + q) + r$

$x = -\frac{1}{2}$ $4 \times \left(-\frac{1}{2}\right)^3 + 5\left(-\frac{1}{2}\right)$ $+ 2 = r$

$x = -\frac{1}{2}$ *used to find a value for r*

(M1)

$r = -5$

(A1)

$p = -1$, $q = 3$

(A1A1)

4

(ii) $\left(\frac{4x^3 + 5x - 2}{2x + 1} = \right) 2x^2 + px + q + \frac{r}{2x + 1}$

M1

$\frac{2}{3}x^3 - \frac{1}{2}x^2 + 3x + k \ln(2x + 1)\ (+ C)$

ft on p and q

A1ft

$\frac{2}{3}x^3 - \frac{1}{2}x^2 + 3x - \frac{5}{2}\ln(2x + 1)(+ C)$

CSO

A1

3

[11]

M8. $u = x^4 + 2$

$\frac{\mathrm{d}u}{\mathrm{d}x} = 4x^3$

or $du = 4x^3dx$

B1

$$\int \frac{x^7}{(x^4+2)^2}\,\mathrm{d}x = \int \frac{k(u-2)}{u^2}\,\mathrm{d}u \text{ or } \int \frac{k(u-2)^{\frac{7}{4}}}{u^2} \frac{\mathrm{d}u}{(u-2)^{\frac{3}{4}}}$$

Either expression all in terms of u including replacing dx, but condone omission of du

M1

$$= \left(\frac{1}{4}\right)\int \frac{1}{u} - \frac{2}{u^2}\,\mathrm{d}u$$

$k \int au^{-1} + bu^{-2}\,du$, *where* k, a, b *are constants*

m1

$$= \left(\frac{1}{4}\right)\left[\ln u + \frac{2}{u}\right]$$

Must have seen du on an earlier line where every term is a term in u

A1

$$= \left(\int = \left(\frac{1}{4}\right)\left[\ln u + \frac{2}{u}\right]_2^3\right)$$

$$\left(\left(\frac{1}{4}\right)\left[\ln\left(x^4+2\right) + \frac{2}{\left(x^4+2\right)}\right]_0^1\right)$$

$= \left(\frac{1}{4}\right)\left[\left(\ln 3+\frac{2}{3}\right)-\left(\ln 2+1\right)\right]$

Dependent on previous A1

Correct change of limits, correct substitution and F(3) − F(2)
or
correct replacement of u, correct substitution and F(1) − F(0)

m1

$= \frac{1}{4}\ln\left(\frac{3}{2}\right)-\frac{1}{12}$

OE in exact form

A1

[6]

M9.(a) $2x + 3 = A(2x + 1) + B(2x - 1)$

M1

$x = \frac{1}{2} \quad x = -\frac{1}{2}$

Use two values of x to find A and B

m1

$A = 2 \quad B = -1$

Both

A1

3

Alternative; equating coefficients

$2x + 3 = A(2x + 1) + B(2x - 1)$

(M1)

x term $2 = 2A + 2B$
constant $3 = A - B$

Set up simultaneous equations and solve.

(m1)

$A = 2 \quad B = -1$

Both

(A1)

(3)

Alternative; cover up rule

$x = \frac{1}{2} \qquad A = \frac{2\times\frac{1}{2}+3}{2\times\frac{1}{2}+1} \qquad \left(=\frac{4}{2}\right)$

$x = \frac{1}{2}$ and $x = -\frac{1}{2}$ used to find A and B

(M1)

$x = -\frac{1}{2} \qquad B = \frac{2\times\left(-\frac{1}{2}\right)+3}{2\times\left(-\frac{1}{2}\right)-1} \qquad \left(=\frac{2}{-2}\right)$

SC NMS

$A = 2 \quad B = -1$

A and B both correct 3 / 3

One of A or B correct 1 / 3

(A1A1)

(3)

Condone poor algebra for M1 if continues correctly.

(b) $4x^2-1\overline{\smash{)}12x^3-7x-6}$ with quotient $3x$

$12x^3 - \underline{3x}$

$-4x - 6$

Complete division leading to values for C and D

M1

$C = 3$

A1

$D = -2$

A1

$C = 3$, $D = -2$ stated or written in expression.

SC B1

$C = 3$, D not found or wrong;

$D = -2$, C not found or wrong.

3

Alternative

$$\frac{12x^3-7x-6}{4x^2-1}=\frac{12x^3-3x-4x-6}{4x^2-1}=3x-\frac{2(2x+3)}{4x^2-1}$$

(M1)

$C = 3$

(A1)

$D = -2$

(A1)

$C = 3$, $D = -2$ stated or written in expression.

SC B1

$C = 3$, D not found or wrong;

$D = -2$, C not found or wrong.

(3)

Alternative

$12x^3 - 7x - 6 = 4Cx^3 - Cx + 2Dx + 3D$

Complete method for C and D

(M1)

$C = 3$

(A1)

$D = -2$

(A1)

C = 3, D = −2 stated or written in expression.

SC B1

C = 3 , D not found or wrong;

D = −2 , C not found or wrong.

(3)

Alternative

$x = 0$ $x = 1$

$6 = -3D$ $-\frac{1}{3} = C + \frac{5}{3}D$

Use two values of x to set up simultaneous equations

(M1)

$C = 3$

(A1)

$D = -2$

(A1)

C = 3, D = −2 stated or written in expression.

SC B1

C = 3 , D not found or wrong;

D = −2 , C not found or wrong.

(3)

Complete division for M1; obtain a value for $C(Cx)$ and a remainder $ax + b$

(c) $\int 3x - 2\left(\frac{2}{2x-1} - \frac{1}{2x+1}\right) \mathrm{d}x$

Use parts (a) and (b) to obtain integrable form

M1

$3\frac{x^2}{2}$

ft on C

A1ft

$-2\left(\ln(2x-1)-\frac{1}{2}\ln(2x+1)\right)$

Both correct; ft on A, B and D

Condone missing brackets

A1ft

$\frac{3}{2}(4-1)-2\left(\left(\ln 3-\frac{1}{2}\ln 5\right)-\left(\ln 1-\frac{1}{2}\ln 3\right)\right)$

Correct substitution of limits

m1

$\frac{9}{2} - 3\ln 3 + \ln 5 = \frac{9}{2} + \ln\left(\frac{5}{27}\right)$

$p = \frac{9}{2}$ $q = \frac{5}{27}$

A1

Form $\int Cx + \left(\frac{P}{2x-1} + \frac{Q}{2x+1}\right)$ *using candidate's P, Q, C for M1.*

Condone missing dx.

$\int Cx \, \mathrm{d}x = C\frac{x^2}{2}$ *for A1ft*

ISW extra terms eg $\frac{12}{4x^2 - 1}$ *for first three terms only; max 3/5.*

Candidate's C*; must have a value.*

$\int \frac{4x+6}{4x^2-1}\,dx = \int \frac{4x}{4x^2-1} + \frac{6}{4x^2-1}$*dx is an integrable form,*

as $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right)$ *is in the formula book, but they **must** try to integrate to show they know this, **or** use partial fractions again with* $\frac{6}{4x^2-1} = \frac{3}{2x-1} - \frac{3}{2x+1}$ *for M1.*

Substitute limits into $C\frac{x^2}{2}$ *+ mln(2x − 1) + nln(2x + 1), or equivalent, for m1; substitution must be completely correct.*

Condone $\frac{9}{2} - \ln\left(\frac{27}{5}\right)$ *for A1.*

5

[11]

M10.(a) (i) $5 - 8x = A(1 - 3x) + B(2 + x)$

M1

$x = -2 \quad x = \frac{1}{3}$

Two values of x used to find values for A *and* B

m1

$A = 3 \quad B = 1$

A1

3

Alternative:

$5 - 8x = A(1 - 3x) + B(2 + x)$

(M1)

$5 = A + 2B$

$-8 = -3A + B$

Set up simultaneous equations and solve.

(m1)

$A = 3 \quad B = 1$

(A1)

(3)

(ii) $\int_{-1}^{0} \frac{3}{2+x} + \frac{1}{1-3x}\, dx = 3 \ln(2 + x) - \frac{1}{3} \ln(1 - 3x)$

a ln(2 + x) + b ln(1 − 3x) where a and b are constants

M1

$= (3 \ln 2 - \frac{1}{3}\ln 1) - (3 \ln 1 - \frac{1}{3} \ln 4)$

f(0) − f(−1) used

m1

$= 3 \ln 2 + \frac{1}{3} \ln 4$

ft A and B

A1ft

$= \frac{11}{3}$ ln 2

ft $\left(A+\frac{2}{3}B\right)$ *ln 2*

A1ft

4

(b) (i) (C =)2

B1

1

(ii) $\int \frac{9-18x-6x^2}{2-5x-3x^2}\,\mathrm{d}x = \int C\,\mathrm{d}x + \int \frac{5-8x}{2-5x-3x^2}\,\mathrm{d}x$

Seen or implied.

Allow $\pm C + \int \frac{5-8x}{2-5x-3x^2}\mathrm{d}x$

M1

$\int_{-1}^{0} \frac{9-18x-6x^2}{2-5x-3x^2}\,\mathrm{d}x = 2+\frac{11}{3}\ln 2$

Accept 2 + 3 ln 2 + $\frac{1}{3}$ *ln 4*

ft 2 + candidate's answer to part (a)(ii) if exact.

A1ft

2

[10]

M11.(a) $\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}=\right)\frac{(\sin\theta\times 0)-1\times\cos\theta}{\sin^2\theta}$

quotient rule $\frac{\pm\sin\theta\times k\pm 1\times\cos\theta}{\sin^2\theta}$ *where* k *= 0 or 1*

M1

must see the '0' either in the quotient or in eg $\frac{\mathrm{d}u}{\mathrm{d}\theta}$ *= 0 etc*

A1

$= -\frac{\cos\theta}{\sin^2\theta}$ or $= -\frac{\cos\theta}{\sin\theta\sin\theta}$

or equivalent

$= -\operatorname{cosec}\theta\cot\theta$

CSO, AG must see one of the previous expressions

A1

3

(b) $x = \operatorname{cosec}\theta$

$\frac{dx}{d\theta} = -\operatorname{cosec}\theta\cot\theta$

OE, eg $dx = -\operatorname{cosec}\theta\cot\theta\, d\theta$

B1

Replacing $\sqrt{(\operatorname{cosec}^2\theta - 1)}$ by $\sqrt{\cot^2\theta}$, or better

at any stage of solution

B1

$\int = \int \frac{-\operatorname{cosec}\theta\cot\theta}{\operatorname{cosec}^2\theta\sqrt{(\operatorname{cosec}^2\theta - 1)}}\, d\theta$

all in terms of θ*, and including their attempt at dx,*
but condone omission of dθ

M1

fully correct and must include dθ *(at some stage in solution)*

A1

$\int \frac{-\operatorname{cosec}\theta \cot\theta}{\operatorname{cosec}^2\theta \cot\theta} (\mathrm{d}\theta) = \int \frac{-1}{\operatorname{cosec}\theta} (\mathrm{d}\theta)$

OE eg $\int -\sin\theta\ (\mathrm{d}\theta)$

A1

= $\cos\theta$

A1

$x = 2$, $\theta = 0.524$ AWRT

$x = \sqrt{2}$, $\theta = 0.785$ AWRT

correct change of limits or (±)cosθ = $(\pm)\left[\sqrt{\left(1-\frac{1}{x^2}\right)}\right]_{\sqrt{2}}^{2}$ *OE*

B1

0.8660 − 0.7071

c's F(0.52) − F(0.79)

substitution into ± cosθ only or $\left(\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\right)$

m1

= 0.159

A1

9

[12]

M12.(a) $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)$ $12x^2 - 6$

do not ISW

B1

1

(b) $\int_2^3 \frac{2x^2-1}{4x^3-6x+1}\,dx = \left[\frac{1}{6}\ln\left(4x^3-6x+1\right)\right]_{(2)}^{(3)}$

k ln ($4x^3$ − 6x + 1), k is a constant

M1

$k = \frac{1}{6}$

A1

$= \frac{1}{6}$ ln (4 × 3^3 − 6 × 3 + 1)

$- \frac{1}{6}$ ln (4 × 2^3 − 6 × 2 + 1)

correct substitution in F(3) − F(2).
condone poor use or lack of brackets.

m1

$= \frac{1}{6}$ ln 91 − $\frac{1}{6}$ ln 21

k ln 91 − k ln 21
only follow through on their k

A1F

$= \frac{1}{6}\ln\frac{91}{21}$ or $\left(= \frac{1}{6}\ln\frac{13}{3}\right)$

A1

or if using the substitution

$u = 4x^3 - 6x + 1$

$\int = k\int\frac{du}{u}$ *M1*

$= \frac{1}{6}$ *ln u* *A1*

then, either change limits to 21 and 91 m1 then A1F A1 as scheme or changing back to 'x', then m1 A1F A1 as scheme

5

[6]

M13.(a) $\int e^{1-2x}\,dx = ke^{1-2x}$ or $e(ke^{-2x})$

where k is a rational number

M1

$$\int_0^{\ln 2} e^{1-2x}dx = -\frac{1}{2}e^{1-2x}\Big|_0^{\ln 2} \text{ or } e\left[-\frac{1}{2}e^{-2x}\right]_0^{\ln 2}$$

correct integration

condone missing limits

A1

$$= \quad = -\frac{1}{2}e^{1-2\ln 2} - -\frac{1}{2}e^{1-2(0)}$$

correct (no decimals)

A1

$$= -\frac{1}{2}\left(\frac{1}{4}e\right) + \frac{1}{2}e$$

eliminating ln

$$= \frac{3}{8}e$$

AG, be convinced

A1

4

(b) $u = \tan x$

$\frac{du}{dx} = \sec^2 x$

PI below, condone $du = \sec^2 x dx$

M1

Replacing dx by $\frac{1}{\sec^2 x}$ (du) in integral

or $\frac{1}{1+u^2}(\mathrm{d}u)$

A1

$\sec^2 x = 1 + u^2$

PI below

B1

$$\left.\begin{array}{l} x = 0 \quad \Rightarrow \quad u = 0 \\ x = \frac{\pi}{4} \quad \Rightarrow \quad u = 1 \end{array}\right\}$$

this could be gained by changing u to tan x after the integration and using x = 0 and $x = \frac{\pi}{4}$

B1

$$\int_0^{\frac{\pi}{4}} \sec^4 x \sqrt{\tan x}\, \mathrm{d}x$$

$= \int (1+u^2)\sqrt{u}\,(\mathrm{d}u)$ or $\int (1+u^2)^2 \sqrt{u} \frac{(\mathrm{d}u)}{1+u^2}$

all in terms of u including replacing dx

all correct, condone omission of du

M1

$= \int \left(u^{\frac{5}{2}} + u^{\frac{1}{2}} \right) \quad (\mathrm{d}u)$

must be in this form

A1

= $\frac{2}{7}u^{\frac{7}{2}}+\frac{2}{3}u^{\frac{3}{2}}$

accept correct unsimplified form

A1

= $\frac{20}{21}$

CAO

A1

8

[12]

M14.(a) $\int x\left(x^2+3\right)^{\frac{1}{2}}\ \mathrm{d}x = p\left(x^2+3\right)^{\frac{3}{2}}$

By inspection or substitution

M1

= $\frac{1}{3}\left(x^2+3\right)^{\frac{3}{2}}$ (+C)

A1

2

(b) $\int \mathrm{e}^{2y}\ \mathrm{d}y = \int x\sqrt{x^2+3}\ \mathrm{d}x$

Correct separation and notation
Condone missing integral signs

B1

$= \frac{1}{2}e^{2y}$

B1

$= \frac{1}{3}\left(x^2+3\right)^{\frac{3}{2}} + C$

Equate to result from (a) with constant.

M1

$\frac{1}{2} = \frac{1}{3} \times 4^{\frac{3}{2}} + C$

Use (1,0) to find constant.

m1

$C = -\frac{13}{6}$

CAO

A1

$2y = \ln\left(\frac{2}{3}\left(x^2+3\right)^{\frac{3}{2}} - \frac{13}{3}\right)$

Solve for y, taking logs correctly.

m1

$y = \frac{1}{2}\ln\left(\frac{2}{3}\left(x^2+3\right)^{\frac{3}{2}} - \frac{13}{3}\right)$

CSO

A1

7

[9]

M15.

	Marking Instructions	**AO**	**Marks**	**Typical Solution**
(a)	Separates variables, at least one side correct.	AO3.1a	M1	$3\sqrt{x\frac{\mathrm{d}x}{\mathrm{d}t}} = 8\sin 2t$
	Obtains correct separation PI	AO1.1b	A1	$\int 3\sqrt{x\frac{\mathrm{d}x}{\mathrm{d}t}}\mathrm{d}t = 8\sin 2t\mathrm{d}t$
	integrates 'their' expressions at least one of 'their' sides correct	AO1.1a	M1	$\int 3x\frac{1}{2}\mathrm{d}x = 8\sin 2t\mathrm{d}t$
	Obtains correct integral (condone missing + c) CAO	AO1.1b	A1	$2x\frac{3}{2} = -4\cos 2t(+c)$
	Substitutes initial conditions, to find + c.	AO3.1b	M1	$2\times(0)^{\frac{3}{2}} = -4\cos(2\times 0)+c$ $c = 4$
	Obtains a correct solution ACF	AO1.1b	A1	$x^{\frac{3}{2}} = 2-2\cos 2t$
	Obtains correct solution of the form $x = \mathrm{f}(t)$	AO2.5	A1	$x = (2-2\cos 2t)^{\frac{2}{3}}$
(b)	Obtains correct max height, in cm Award FT from correct substitution into incorrect equation $x = \mathrm{f}(t)$ but only if all three M1 marks have been awarded, must have correct units.	AO3.4	A1F	max height $= 4^{\frac{2}{3}} = 252$ cm
				Total 8 marks

M16.(a) $h = 1$

PI

B1

f(x) = $\frac{1}{x^2+1}$

I ≈ $\frac{h}{2}$ {f(1) + f(5) + 2[f(2) + f(3) + f(4)]}

$\frac{h}{2}$ {f(1) + f(5) + 2[f(2) + f(3) + f(4)]}

OE summing of areas of the four 'trapezia'…

M1

$\frac{h}{2}$ with $\{\ldots\} = \frac{1}{2} + \frac{1}{26} + 2\left(\frac{1}{5} + \frac{1}{10} + \frac{1}{17}\right)$

= 0.5 + 0.03(84...) + 2[0.2+0.1+0.05(88...)]
= 0.538(46…) + 2[0.358(82...)] = 1.256(108…)

OE Accept 2dp (rounded or truncated) for non-terminating decs. equiv.

A1

(I ≈) 0.628054… = $\frac{694}{1105}$ = 0.628 (to 3sf)

CAO Must be 0.628

***SC** for those who use 5 strips, max possible is B0M1A1A0*

A1

4

(b) (i) $\int \left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}} \right) dx = \frac{x^{-\frac{1}{2}}}{-1/2} + \frac{6x^{\frac{3}{2}}}{3/2}$ (+ c)

One term correct (even unsimplified)

M1

Both terms correct (even unsimplified)

A1

$= -2x^{-0.5} + 4x^{1.5}$ (+ c)

Must be simplified

A1

3

(ii) $\int_1^4 \left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}} \right) \mathrm{d}x = [-2(4^{-0.5}) + 4(4^{1.5})] - [-2(1^{-0.5}) + 4(1^{1.5})]$

Attempt to calculate F(4) − F(1) where F(x) follows integration and is not just the integrand

M1

$= (-1 + 32) - (-2 + 4) = 29$

Since 'Hence' NMS scores 0 / 2

A1

2

[9]

M17.

Marking Instructions	**AO**	**Marks**	**Typical Solution**
Selects a method of integration, which could lead to a correct solution. Evidence of integration by parts OR an attempt at integration by inspection.	AO3.1a	M1	$u = \ln 2x; \quad \frac{\mathrm{d}v}{\mathrm{d}x} = x^3$ $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}; \quad v = \frac{x^4}{4}$ $\left[\frac{x4}{4}\ln(2x)\right]_1^2 - \int_1^2 \frac{x^3}{4}\mathrm{d}x$ $\left[\frac{x4}{4}\ln(2x) - \frac{x^4}{16}\right]_1^2$ $= \left(\frac{2^4}{4}\ln(4) - \frac{2^4}{16}\right) - \left(\frac{1}{4}\ln(2) - \frac{1}{16}\right)$
Applies integration by parts formula correctly OR correctly differentiates an expression of the form $Ax^4\ln 2x$	AO1.1b	A1	
Obtains correct integral, condone missing limits.	AO1.1b	A1	

Substitutes correct limits into 'their' integral	AO1.1a	M1	$\frac{31}{4}\ln 2 - \frac{15}{16}$ so $p = \frac{31}{4}$ $q = -\frac{15}{16}$ **ALT** $\frac{d}{dx}(x^4\ln 2x) = 4x^3\ln 2x + x^4.\frac{1}{x}$ $\therefore \int_1^2 x^3\ln 2x dx = \left[\frac{1}{4}(x^4\ln 2x - \frac{x^4}{4})\right]_1^2$ $= \left(\frac{2^4}{4}\ln(4) - \frac{2^4}{16}\right) - \left(\frac{1}{4}\ln(2) - \frac{1}{16}\right)$ $\frac{31}{4}\ln 2 - \frac{15}{16}$ $p = \frac{31}{4}$ $q = -\frac{15}{16}$
Obtains correct p and q FT use of incorrect integral provided both M1 marks have been awarded	AO1.1b	A1F	
			Total 5 marks