P3 Vectors

Revision Questions

Name: ____________________

Class: ____________________

Date: ____________________

Time:	**132 minutes**
Marks:	**96 marks**
Comments:	

1. With respect to the origin O, the points A and B have position vectors given by

$$\overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = \mathbf{i} + 4\mathbf{j} + 3\mathbf{k}.$$

The line l has vector equation $\mathbf{r} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} + s(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$.

(i) Prove that the line l does not intersect the line through A and B.

[5]

(ii) Find the equation of the plane containing l and the point A, giving your answer in the form $ax + by + cz = d$.

[6]

2. The straight line l passes through the points A and B with position vectors

$$2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \qquad \text{and} \qquad \mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$$

respectively. This line intersects the plane p with equation $x - 2y + 2z = 6$ at the point C.

(i) Find the position vector of C.

[4]

(ii) Find the acute angle between l and p.

[4]

(iii) Show that the perpendicular distance from A to p is equal to 2.

[3]

3. The points A and B have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \text{ and } \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}.$$

The line l passes through A and is parallel to OB. The point N is the foot of the perpendicular from B to l.

(i) State a vector equation for the line l. **[1]**

(ii) Find the position vector of N and show that $BN = 3$. **[6]**

(iii) Find the equation of the plane containing A, B and N, giving your answer in the form $ax + by + cz = d$. **[5]**

4. The line l has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} - \mathbf{k} + t(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$. It is given that l lies in the plane with equation $2x + by + cz = 1$, where b and c are constants.

(i) Find the values of b and c.

[6]

(ii) The point P has position vector $2\mathbf{j} + 4\mathbf{k}$. Show that the perpendicular distance from P to l is $\sqrt{5}$.

[5]

5. Two planes have equations $2x - y - 3z = 7$ and $x + 2y + 2z = 0$.

(i) Find the acute angle between the planes.

[4]

(ii) Find a vector equation for their line of intersection.

[6]

6. The line l has equation $\mathbf{r} = \mathbf{j} + \mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. The plane p has equation $x + 2y + 3z = 5$.

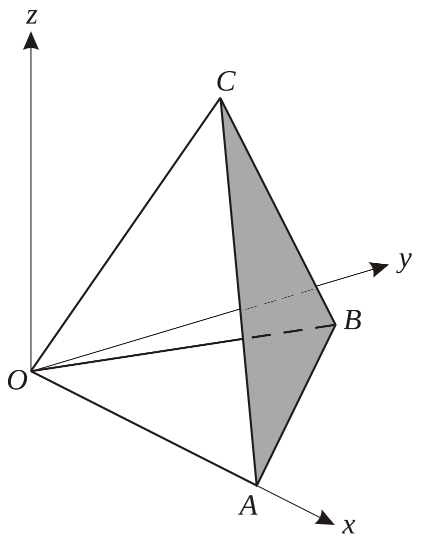
(i) Show that the line l lies in the plane p.

[3]

(ii) A second plane is perpendicular to the plane p, parallel to the line l and contains the point with position vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$. Find the equation of this plane, giving your answer in the form $ax + by + cz = d$.

[6]

7.



The diagram shows a set of rectangular axes *Ox*, *Oy* and *Oz*, and three points *A*, *B* and *C* with position vectors $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.

(i) Find the equation of the plane *ABC*, giving your answer in the form $ax + by + cz = d$. **[6]**

(ii) Calculate the acute angle between the planes *ABC* and *OAB*. **[4]**

8. The points A and B have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}.$$

The line l has vector equation

$$\mathbf{r} = (1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}.$$

(i) Show that l does not intersect the line passing through A and B. **[4]**

(ii) The point P lies on l and is such that angle PAB is equal to 60°. Given that the position vector of P is $(1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}$, show that $3t^2 + 7t + 2 = 0$. Hence find the only possible position vector of P. **[6]**

9. The straight line l has equation $\mathbf{r} = \mathbf{i} + 6\mathbf{j} - 3\mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$. The plane p has equation $(\mathbf{r} - 3\mathbf{i}).(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) = 0$. The line l intersects the plane p at the point A.

(i) Find the position vector of A.

[3]

(ii) Find the acute angle between l and p.

[4]

(iii) Find a vector equation for the line which lies in p, passes through A and is perpendicular to l.

[5]

1.	(i)	State or imply a direction vector for *AB* is –**i** + 2**j** + 2**k**, or equivalent	B1
		EITHER:	
		State equation of *AB* is **r** = 2**i** + 2**j** + **k** +*t*(–**i** + 2**j** + 2**k**), or equivalent	B1√
		Equate at least two pairs of components of *AB* and *l* and solve for *s* or for *t*	M1
		Obtain correct answer for *s* or for t, e.g. $s = 0$ or $t = -2$; $s = -\frac{5}{3}$ **or** $t = -\frac{1}{3}$ or $s = 5$ or $t = 3$	A1
		Verify that all three pairs of equations are not satisfied and that the lines fail to intersect	A1
		OR:	
		State a Cartesian equation for *AB*, e.g. $\frac{x-2}{-1}=\frac{y-2}{2}=\frac{z-1}{2}$, and for *l*, e.g. $\frac{x-4}{1}=\frac{y+2}{2}=\frac{z-2}{1}$	B1√
		Solve a pair of equations, e.g. in *x* and *y*, for one unknown	M1
		Obtain one unknown, e.g. $x = 4$ or $y = –2$	A1
		Obtain corresponding remaining values, e.g. of *z*, and show lines do not intersect	A1
		OR:	
		Form a relevant triple scalar product, e.g. (2**i** – 4**j** + **k**).((–**i** + 2**j** + 2**k**)×(**i** + 2**j** + **k**))	B1√
		Attempt to use correct method of evaluation	M1
		Obtain at least two correct simplified terms of the three terms of the complete expansion of the triple product or of the corresponding determinant	A1
		Obtain correct non-zero value, e.g. – 20, and state that the lines do not intersect	A1 5
	(ii)	**EITHER:**	
		Obtain a vector parallel to the plane and not parallel to *l*, e.g. 2**i** – 4**j** + **k**	B1
		Use scalar product to obtain an equation in *a*, *b* and *c*, e.g. $a + 2b + c = 0$	B1
		Form a second relevant equation, e.g. $2a – 4b + c = 0$ and solve for one ratio, e.g. *a* : *b*	M1

Obtain final answer $a : b : c = 6 : 1 : –8$	A1
Use coordinates of a relevant point and values of a, b and c in general equation and find d	M1
Obtain answer $6x + y – 8z = 6$, or equivalent	A1
OR:	
Obtain a vector parallel to the plane and not parallel to l, e.g. $2\mathbf{i} – 4\mathbf{j} + \mathbf{k}$	B1
Obtain a second relevant vector parallel to the plane and attempt to calculate their vector product, e.g. $(\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \times (2\mathbf{i} – 4\mathbf{j} + \mathbf{k})$	M1
Obtain two correct components of the product	A1
Obtain correct answer, e.g. $6\mathbf{i} + \mathbf{j} – 8\mathbf{k}$	A1
Substitute coordinates of a relevant point in $6x + y – 8z = d$, or equivalent, to find d	M1
Obtain answer $6x + y – 8z = 6$, or equivalent	A1
OR:	
Obtain a vector parallel to the plane and not parallel to l, e.g. $2\mathbf{i} – 4\mathbf{j} + \mathbf{k}$	B1
Obtain a second relevant vector parallel to the plane and correctly form a 2-parameter equation for the plane, e.g. $\mathbf{r} = 2\mathbf{i} +2\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} – 4\mathbf{j} +\mathbf{k}) +\mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	M1
State 3 correct equations in x, y, z, λ and μ	A1
Eliminate λ and μ	M1
Obtain equation in any correct form	A1
Obtain answer $6x + y – 8z = 6$, or equivalent	A1
OR:	
Using the coordinates of A and two points on l, state three simultaneous equations in a, b, c and d, e.g. $2a + 2b + c = d$, $4a – 2b + 2c = d$ and $5a + 3c = d$	B1
Solve and find one ratio, e.g. $a : b$	M1
State one correct ratio	A1
Obtain a ratio of three unknowns, e.g. $a : b : c = 6 : 1 : –8$, or equivalent	A1
Either use coordinates of a relevant point and found ratio to find fourth unknown, d, **or** find the ratio of all four unknowns	M1
Obtain answer $6x + y – 8z = 6$, or equivalent	A16
	[11]

2.	(i)	State or imply a direction vector of *AB* is $-\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, or equivalent	B1	
		State equation of *AB* is $\mathbf{r} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, or equivalent	B1√	
		Substitute in equation of *p* and solve for λ	M1	
		Obtain $4\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ as position vector of *C*	A1	4
	(ii)	State or imply a normal vector of *p* is $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, or equivalent	B1	
		Carry out correct process for evaluating the scalar product of two relevant vectors, e.g. $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}).(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$	M1	
		Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine or inverse sine of the result	M1	
		Obtain answer 24.1°	A1	4
	(iii)	**EITHER:**		
		Obtain *AC* (= $\sqrt{24}$) in any correct form	B1√	
		Use trig to obtain length of perpendicular from *A* to *p*	M1	
		Obtain given answer correctly	A1	
		OR:		
		State or imply $\overrightarrow{AC}$ is $2\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$, or equivalent	B1√	
		Use scalar product of $\overrightarrow{AC}$ and a unit normal of *p* to calculate the perpendicular	M1	
		Obtain given answer correctly	A1	
		OR:		
		Use plane perpendicular formula to find perpendicular from *A* to *p*	M1	
		Obtain a correct unsimplified numerical expression, e.g. $\dfrac{\lvert 2-2(2)+2(1)-6\rvert}{\sqrt{(1^2+(-2)^2+2^2)}}$	A1	
		Obtain given answer correctly	A1	3

[11]

3.	(i)	State $\mathbf{r} = \begin{pmatrix}-1\\3\\5\end{pmatrix} + \lambda\begin{pmatrix}3\\-1\\-4\end{pmatrix}$, or equivalent	B1	1
	(ii)	Express $\overrightarrow{BN}$ in terms of λ, e.g. $\begin{pmatrix}-1+3\lambda\\3-\lambda\\5-4\lambda\end{pmatrix} - \begin{pmatrix}3\\-1\\-4\end{pmatrix}$, or equivalent	B1	
		Equate its scalar product with $\begin{pmatrix}3\\-1\\-4\end{pmatrix}$ to zero and solve for λ	M1	

Obtain $\lambda = 2$ A1

Obtain $\overrightarrow{ON} = \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix}$, or equivalent A1√

Carry out method for calculating BN, i.e. $|2\,\mathbf{i} + 2\mathbf{j} + \mathbf{k}|$ M1

Obtain the given answer $BN = 3$ correctly A16

(iii) ***EITHER:***

Use scalar product to obtain a relevant equation in a, b and c, e.g. $3a - b - 4c = 0$ or $2a + 2b + c = 0$	M1
Sate two correct equations in a, b, c	A1√
Solve simultaneous equations to obtain one ratio, e.g. $a : b$	M1
Obtain $a : b : c = 7 : -11 : 8$, or equivalent	A1
Obtain equation $7x - 11y + 8z = 0$, or equivalent	A1

OR:

Substitute for A, B and N in equation of plane and state 3 equations in a, b, c, d	B1
Eliminate one unknown, e.g. d, entirely and obtain 2 equation in 3 unknowns	M1
Solve to obtain one ratio e.g. $a : b$	M1
Obtain $a : b : c = 7: -11 : 8$, or equivalent	A1
Obtain equation $7x - 11y + 8z = 0$, or equivalent	A1

OR:

Calculate vector product of two vector parallel to the plane, e.g. $(3\mathbf{i} - \mathbf{j} - 4\mathbf{k}) \times (2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	M1
Obtain 2 correct components of the product	A1√
Obtain correct product e.g. $7\mathbf{i} - 11\mathbf{j} + 8\mathbf{k}$, or equivalent	A1
Substitute in $7x - 11y + 8z = d$ and find d, or equivalent	M1
Obtain equivalent $7x - 11y + 8z = 0$, or equivalent	A1

OR:

Form correctly a 2-parameter equation for the plane	M1	
Obtain equation in any correct form e.g. $\mathbf{r} = -\mathbf{i} + 3\mathbf{j} + 5\mathbf{k} + \lambda(3\mathbf{i} - \mathbf{j} - 4\mathbf{k}) + \mu(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	A1√	
State 3 equation in x, y, z, λ, and μ	A1	
Eliminate λ and μ	M1	
Obtain equation $7x - 11y + 8z = 0$, or equivalent	A1	5

[12]

4. (i) **EITHER**

Substitute coordinates of general point of l in equation of plane and equate constant terms, obtaining an equation in b and c — M1*

Obtain a correct equation, e.g. $8 + 2b - c = 1$ — A1

Equate the coefficient of t to zero, obtaining an equation in b and c — M1*

Obtain a correct equation, e.g. $4 - b - 2c = 0$ — A1

OR

Substitute $(4, 2, -1)$ in the plane equation — M1*

Obtain a correct equation in b and c, e.g. $2b - c = -7$ — A1

EITHER

Find a second point on l and obtain an equation in b and c — M1*

Obtain a correct equation in b and c, e.g. $b + 2c = 4$ — A1

OR

Calculate scalar product of a direction vector for l and a vector normal for the plane and equate to zero — M1*

Obtain a correct equation for b and c — A1

Solve for b or for c — M1 (dep*)

Obtain $b = -2$ and $c = 3$ — A1 6

(ii) **EITHER**

Find $\overrightarrow{PQ}$ for a point Q on l with parameter t, e.g. $4\mathbf{i} - 5\mathbf{k} + t(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ — B1

Calculate scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate to zero — M1

Solve and obtain $t = -2$ — A1

Carry out a complete method for finding the length of $\overrightarrow{PQ}$ — M1

Obtain the given answer $\sqrt{5}$ correctly — A1

OR 1

Calling $(4, 2, -1)$ A, state $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form, e.g. $4\mathbf{i} - 5\mathbf{k}$ — B1

Calculate vector product of $\overrightarrow{AP}$ and a direction vector for l, e.g. $(4\mathbf{i} - 5\mathbf{k}) \times (2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ — M1

Obtain correct answer, e.g. $-5\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$ — A1

Divide modulus of the product by that of the direction vector M1

Obtain the given answer correctly A1

OR 2

State $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form B1

Use a scalar product to find the projection of $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) on l M1

Obtain correct answer in any form, e.g. $\dfrac{18}{\sqrt{9}}$ A1

Use Pythagoras to find the perpendicular M1

Obtain the given answer correctly A1

OR 3

State $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form B1

Use a scalar product to find the cosine of *PAQ* M1

Obtain correct answer in any form, e.g. $\dfrac{18}{\sqrt{41}\cdot\sqrt{9}}$ A1

Use trig to find the perpendicular M1

Obtain the given answer correctly A1

OR 4

State $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form B1

Find a second point *B* on *l* and use the cosine rule in triangle *APB* to find the cosine of *A*, *B* or *P*, or use a vector product to find the area of *APB* M1

Obtain correct answer in any form A1

Use trig or area formula to find the perpendicular M1

Obtain the given answer correctly A1

OR 5

Find $\overrightarrow{PQ}$ for a point *Q* on *l* with parameter t, e.g. $4\mathbf{i} - 5\mathbf{k} + t(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ B1

Use correct method to express PQ^2 (or PQ) in terms of t M1

Obtain a correct expression in any form, e.g. $(4 + 2t)^2 + (-t)^2 + (-5 - 2t)^2$ A1

		Carry out a complete method for finding its minimum	M1
		Obtain the given answer correctly	A1 5 **[11]**
5.	(i)	State or imply a correct normal vector to either plane, e.g. 2**i** – **j** – 3**k**, or **i** + 2**j** + 2**k**	B1
		Carry out correct process for evaluating the scalar product of the two normals	M1
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	M1
		Obtain answer 57.7° (or 1.01 radians)	A1 4
	(ii)	***EITHER:***	
		Carry out a complete method for finding a point on the line	M1
		Obtain such a point, e.g. (2, 0, –1)	A1

EITHER:

State two correct equations for a direction vector of the line, e.g. $2a - b - 3c = 0$ and $a + 2b + 2c = 0$	B1
Solve for one ratio, e.g. $a : b$	M1
Obtain $a : b : c = 4 : -7 : 5$, or equivalent	A1
State a correct answer, e.g. **r** = 2**i** – **k** + λ(4**i** – 7**j** + 5**k**)	A1√

OR:

Obtain a second point on the line, e.g. $(0, \frac{7}{2}, -\frac{7}{2})$	A1
Subtract position vectors to obtain a direction vector	M1
Obtain 4**i** – 7**j** + 5**k**, or equivalent	A1
State a correct answer, e.g. **r** = 2**i** – **k** + λ(4**i** – 7**j** + 5**k**)	A1√

OR:

Attempt to calculate the vector product of two normals	M1
Obtain two correct components	A1
Obtain 4**i** – 7**j** + 5**k**, or equivalent	A1
State a correct answer, e.g. **r** = 2**i** – **k** + λ(4**i** – 7**j** + 5**k**)	A1√

***OR*1**

Express one variable in terms of a second	M1
Obtain a correct simplified expression, e.g. $x = \frac{14-4y}{7}$	A1

Express the first variable in terms of a third M1

Obtain a correct simplified expression, e.g. $x = \frac{14-4z}{5}$ A1

Form a vector equation for the line M1

State a correct answer, e.g. $\mathbf{r} = \frac{7}{2}\mathbf{j} - \frac{7}{2}\mathbf{k} + \lambda(\mathbf{i} - \frac{7}{4}\mathbf{j} + \frac{5}{4}\mathbf{k})$,
or equivalent A1√

OR2:

Express one variable in terms of a second M1

Obtain a correct simplified expression, e.g. $y = \frac{14-7x}{4}$ A1

Express the third variable in terms of the second M1

Obtain a correct simplified expression, e.g. $z = \frac{5x-14}{4}$ A1

Form a vector equation for the line M1

State a correct answer, e.g. $\mathbf{r} = \frac{7}{2}\mathbf{j} - \frac{7}{2}\mathbf{k} + \lambda(\mathbf{i} - \frac{7}{4}\mathbf{j} + \frac{5}{4}\mathbf{k})$,
or equivalent A1√6

[The f.t. is dependent on all M marks having been obtained.]

[10]

6. (i) ***EITHER:***

State or imply general point of l has coordinates $(x, 1 - 2x, 1 + x)$, or equivalent	B1	
Substitute in LHS of plane equation	M1	
Verify that the equation is satisfied	A1	

OR:

State or imply the plane has equation **r**, (**i** + 2**j** + 3**k**) = 5, or equivalent	B1	
Substitute for **r** in LHS and expand the scalar product	M1	
Verify that the equation is satisfied	A1	

OR:

Verify that a point of l lies on the plane	B1	
Find a second point on l and substitute its coordinates in the equation of p	M1	
Verify second point, e.g. (1,–1, 2) lies on the plane	A1	

OR:

Verify that a point of l lies on the plane	B1	
Form scalar product of a direction vector of l with a vector normal to p	M1	
Verify scalar product is zero and l is parallel to p	A1	3

(ii) ***EITHER:***

Use scalar product of relevant vectors to form an equation in a, b, c, e.g. $a - 2b + c = 0$ or $a + 2b + 3c = 0$	M1*	
State two correct equations in a, b, c	A1	
Solve simultaneous equations and find one ration, e.g., $a : b$	M1(dep*)	
Obtain $a : b : c = 4 : 1 : -2$, or equivalent	A1	
Substitute correctly in $4x + y - 2 - d$ to find d	M1	
Obtain equation $4x + y - 2z - 1$, or equivalent	A1	

OR:

Attempt to calculate vector product of relevant vectors, e.g. (**i** – 2**j** + **k**) × (**i** + 2**j** + 3**k**)	M2	
Obtain 2 correct components of the product	A1	
Obtain correct products, e.g. –8**i** – 2**j** + 4**k**	A1	
Substitute correctly in $4x + y - 2z = d$ to find d	M1	
Obtain equation $4x + y - 2z = 1$, or equivalent	A1	
[SR: If the outcome of the vector product is the negative of the correct answer allow the final mark to be available, i.e. M2A0A0M1A1 is possible]		

OR:

Attempt to form 2-parameter equation for the plane with relevant vectors	M2	

	State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + \mu(\mathbf{I} + 2\mathbf{j} + 3\mathbf{k})$	A1	
	State 3 equations in x, y, λ, μ	A1	
	Eliminate λ and μ	M1	
	Obtain equation $4x + y - 2z - 1$, or equivalent	A1	6
			[9]

7. (i) ***EITHER*:**

Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = -\mathbf{i} + 2\mathbf{j}$	B1
Use scalar product of perpendicular vector to obtain an equation in a, b, c, e.g. $-a + 2b = 0$ or $-a + b + 2c = 0$, or $-b + 2c = 0$	M1
Obtain two correct equations in a, b, c	A1
Solve to obtain ratio $a: b: c$, or equivalent	M1
Obtain $a : b: c = 4 : 2 : 1$, or equivalent	A1
Obtain equivalent $4x + 2y + z = 8$, or equivalent	A1

OR1:

Substitute for A and B and obtain $2a = d$ and $a + 2b = d$	B1
Substitute for C to obtain a third equation and eliminate one unknown (a, b, or d) entirely	M1
Obtain two correct equation in three unknowns e.g. a, b, c	A1
Solve to obtain their ratio, e.g. $a : b : c$, or equivalent	M1
Obtain $a: b : c = 4 : 2: 1$, or $a : c: d = 4 : 1: 8$ or $b : c: d = 2 : 1: 8$, or equivalent	A1
Obtain equation $4x + 2y + z = 8$, or equivalent	A1

OR2:

Substitute for A and B and obtain $2a - d$ and $a + 2b = d$	B1
Solve to obtain ratio $a: b: d$, or equivalent	M2
Obtain $a: b: d = 2: 1: 4$, or equivalent	A1
Substitute for C to find c	M1
Obtain equation $4x + 2y + z = 8$, or equivalent	A1

OR3:

Obtain a vector parallel to the plane e.g. $\overrightarrow{BC} = -\mathbf{j} + 2\mathbf{k}$	B1
Obtain a second such vector and calculate their vector product, e.g. $(-\mathbf{i} + 2\mathbf{j}) \times (-\mathbf{j} + 2\mathbf{k})$	M1
Obtain two correct components of the product	A1
Obtain correct answer, e.g. $4\mathbf{i} + 2\mathbf{j} + \mathbf{k}$	A1
Substitute in $4x + 2y + z = d$ to find d	M1
Obtain equation $4x + 2y + z = 8$, or equivalent	A1

OR4:

Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	B1

		Obtain a second such vector and form correctly a 2-parameter equation for the plane	M1	
		Obtain a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + \lambda(-\mathbf{i} + 2\mathbf{j}) + \mu(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	A1	
		State three equations in x, y, z, λ, μ	A1	
		Eliminate λ and μ	M1	
		Obtain equation $4x + 2y + z = 8$, or equivalent	A1	6
	(ii)	State or imply a normal vector for plane OAB is $\mathbf{k}$, or equivalent		B1
		Carry out correct process for evaluating a scalar product of two relevant vectors, $(4\mathbf{i} + 2\mathbf{j} + \mathbf{k}).(\mathbf{k})$		M1
		Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result		M1
		Obtain answer 77.4° or 1.35 radians		A14
				[10]
8.	(i)	State a vector equation for the line through A and B, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + s(\mathbf{i} - \mathbf{j})$	B1	
		Equate at least two pairs of components of general points on AB and l, and solve for s or for t		M1
		Obtain correct answer for s or t, e.g. $s = -6, 2, -2$ when $t = 3, -1, -1$ respectively		A1
		Verify that all three component equations are not satisfied		A14
	(ii)	State or imply a direction vector for AP has components $(-2t, 3 + t, -1 - t)$, or equivalent		B1
		State or imply cos 60° equals $\dfrac{\overrightarrow{AP}.\overrightarrow{AB}}{\lvert\overrightarrow{AP}\rvert.\lvert\overrightarrow{AB}\rvert}$		M1*
		Carry out correct processes for expanding the scalar product and expressing the product of the moduli in terms of t, in order to obtain an equation in t in any form		M1(dep*)
		Obtain the given equation $3t^2 + 7t + 2 = 0$ correctly		A1
		Solve the quadratic and use a root to find a position vector for P		M1
		Obtain position vector $5\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ from $t = -2$, having rejected the root $t = -\frac{1}{3}$ for a valid reason		A16
				[10]

9. (i) Substitute for r and expand the given scalar product, or correct equivalent, to obtain an equation in s — M1

Solve a linear equation formed from a scalar product for s — M1

Obtain $s = 2$ and position vector $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ for A — A1 3

(ii) State or imply a normal vector of p is $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$, or equivalent — B1

Use the correct process for evaluating a relevant scalar product, e.g. $(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}).(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$ — M1

Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result — M1

Obtain final answer 72.2° or 1.26 radians — A1 4

(iii) ***EITHER:***

Taking the direction vector of the line to be $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, state equation $2a - 3b + 6c = 0$	B1
State equation $a - 2b + 2c = 0$	B1
Solve to find one ratio, e.g. $a : b$	M1
Obtain ratio $a : b : c = 6 : 2 : -1$, or equivalent	A1
State answer $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(6\mathbf{i} + 2\mathbf{j} - \mathbf{k})$, or equivalent	A1√

OR1:

Attempt to calculate the vector product of a direction vector for the line l and a normal vector of the plane p, e.g.$(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) \times (2\mathbf{i} -3\mathbf{j} + 6\mathbf{k})$	M2
Obtain two correct components of the product	A1
Obtain answer $-6\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, or equivalent	A1
State answer $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(-6\mathbf{i} - 2\mathbf{j} + \mathbf{k})$, or equivalent	A1√

OR2:

Obtain the equation of the plane containing A and perpendicular to the line l	M1	
State answer $x -2y + 2z = 1$, or equivalent	A1√	
Find position vector of a second point B on the line of intersection of this plane with the plane p, e.g. $9\mathbf{i} + 4\mathbf{j}$	M1	
Obtain a direction vector for this line of intersection, e.g. $6\mathbf{i} + 2\mathbf{j} - \mathbf{k}$	A1	
State answer $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(6\mathbf{i} + 2\mathbf{j} - \mathbf{k})$, or equivalent	A1	5

[The f.t. is on A.]

[12]