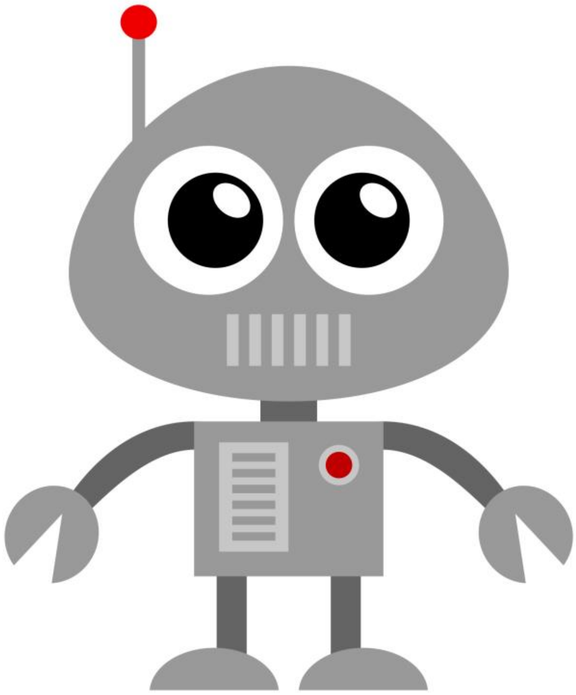


Polynomials, Modulus and Logarithm Practice Test

1. The polynomial $x^3 - x^2 + ax + b$ is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x - 2)$ the remainder is 12.

(i) Find the values of a and b.

[5]

(ii) When a and b have these values, factorise $p(x)$.

[2]

2. Solve the equation $12 = |3x - 2|$.

[2]

3. (i) Solve the equation $2^x + 1 = 45$ giving your answer to 3SF.

[1]

(ii) Solve the inequality $(0.8)^x < 0.5$ giving your answer to 3SF

[2]

(iii) Find the exact solution to the equation $\log_6(x + 5) + \log_6 x = 2$

[3]

4. (i) Sketch the graphs of $y = |2x - 1|$ and $y = |x|$ on the same set of axes.

[3]

(ii) Solve the inequality $|2x - 1| > |x|$.

[3]

5. The polynomial $2x^3 - 3x^2 + ax + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x + 2)$ the remainder is -20.

(i) Find the values of a and b.

[5]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by $(x^2 - 4)$.

[3]

6. (i) Express 4^x in terms of y, where $y = 2^x$.

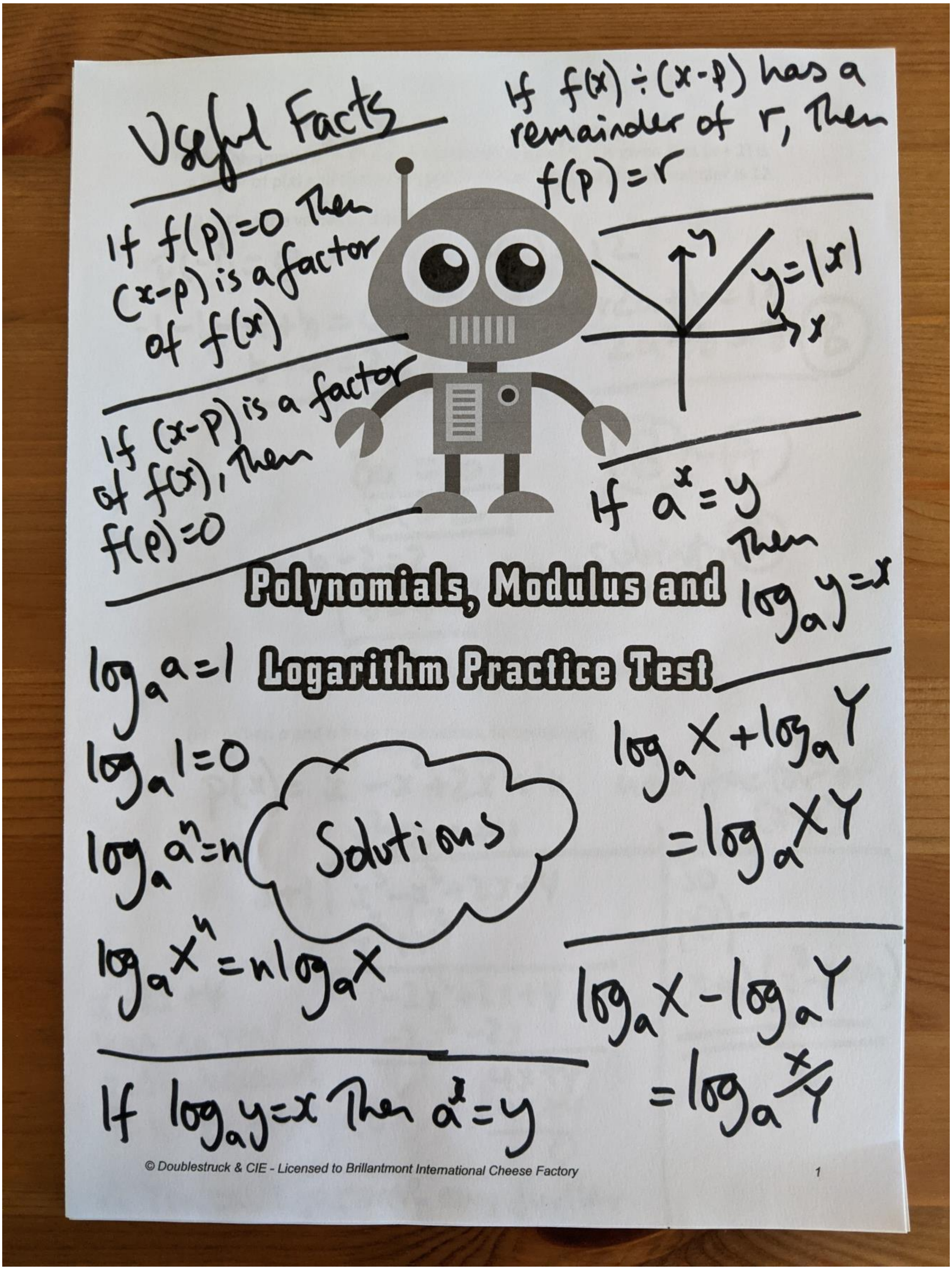
[1]

(ii) Hence find the values of x that satisfy the equation

$$3(4^x) - 10(2^x) + 3 = 0,$$

giving your answers correct to 2 decimal places.

[5]


Useful Facts
If f(x) ÷ (x-p) has a remainder of r, Then f(p) = r
If f(p)=0 Then (x-p) is a factor of f(x)
y=|x|
y
x
If (x-p) is a factor of f(x), Then f(p)=0
If a^x = y Then log_a y = x
Polynomials, Modulus and Logarithm Practice Test
log_a a = 1
log_a 1 = 0
log_a a^n = n
Solutions
log_a X + log_a Y = log_a XY
log_a x^n = n log_a x
log_a X - log_a Y = log_a X/Y
If log_a y = x Then a^x = y
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1

1. The polynomial $x^3 - x^2 + ax + b$ is denoted by p(x). It is given that (x + 1) is a factor of p(x) and that when p(x) is divided by (x – 2) the remainder is 12.

(i) Find the values of a and b. **[5]**

$p(-1) = 0$

$-1 - 1 - a + b = 0$

$b - a = 2$ (A)

$p(2) = 12$

$8 - 4 + 2a + b = 12$

$2a + b = 8$ (B)

(B) − (A)

$3a = 6$

$a = 2$

subinto (A)

$b - 2 = 2$

$b = 4$

(ii) When a and b have these values, factorise p(x). **[2]**

$p(x) = x^3 - x^2 + 2x + 4$ has factor of $(x+1)$

$$\begin{array}{r|l} & x^2 - 2x + 4 \\ x+1 & x^3 - x^2 + 2x + 4 \\ & x^3 + x^2 \\ \hline & -2x^2 + 2x + 4 \\ & -2x^2 - 2x \\ \hline & 4x + 4 \\ & 4x + 4 \\ \hline & 0 \end{array}$$

So $p(x) = (x+1)(x^2 - 2x + 4)$

$x^2 - 2x + 4$ has no real roots because $b^2 - 4ac = 4 - 4(4) < 0$

∴ You can't factorise any further.

2. Solve the equation $12 = |3x - 2|$. **[2]**

$+$ / $-$

$3x - 2 = 12$

$3x = 14$

$x = \frac{14}{3}$

$-3x + 2 = 12$

$-3x = 10$

$x = \frac{-10}{3}$

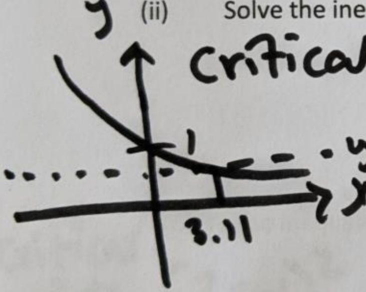
3. (i) Solve the equation $2^x + 1 = 45$ giving your answer to 3SF. **[1]**

$2^x = 44$

$\log_2 44 = x$

$x = \underline{5.46}$ (3SF)

(ii) Solve the inequality $(0.8)^x < 0.5$ giving your answer to 3SF **[2]**



critical point when $0.8^x = 0.5$

$x \log 0.8 = \log 0.5$

$x = 3.11$ (3SF)

$x > 3.11$ (3SF)

(iii) Find the exact solution to the equation $\log_6(x + 5) + \log_6 x = 2$ **[3]**

$\log_6(x^2 + 5x) = 2$

$36 = x^2 + 5x$

$0 = x^2 + 5x - 36$

$0 = (x + 9)(x - 4)$

~~$x = -9$~~ or ONLY $x = 4$

Not a legitimate solution because $\log_6(-9+5)$ has no value.

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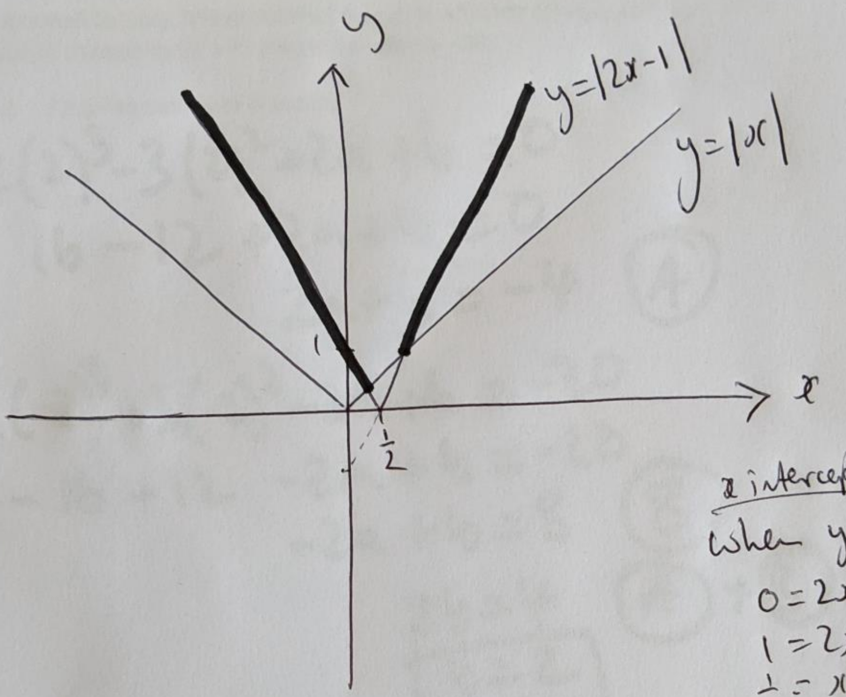
4. (i) Sketch the graphs of y=|2*x* – 1| and y=|*x*| on the same set of axes. **[3]**



x intercept of y=|2x-1|

when y=0

$0 = 2x - 1$

$1 = 2x$

$\frac{1}{2} = x$

(ii) Solve the inequality |2*x* – 1| > |*x*|. **[3]**

critical points $(2x-1)^2 = x^2$

$4x^2 - 4x + 1 = x^2$

$3x^2 - 4x + 1 = 0$

$(3x-1)(x-1) = 0$

$x = \frac{1}{3}$ OR $x = 1$

So $x > 1$

OR $x < \frac{1}{3}$

Use graph to describe the regions!

5. The polynomial $2x^3 - 3x^2 + ax + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x + 2)$ the remainder is -20.

(i) Find the values of a and b. **[5]**

$2(2)^3 - 3(2)^2 + 2a + b = 0$

$16 - 12 + 2a + b = 0$

$2a + b = -4$ (A)

$2(-2)^3 - 3(-2)^2 - 2a + b = -20$

$-16 - 12 \quad -2a + b = -20$

$-2a + b = 8$ (B)

$2b = 4$ (A) + (B)

$b = 2$

$a = -3$

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by $(x^2 - 4)$. **[3]**

$$
\begin{array}{r|l}
 & 2x \ -3 \\
x^2 - 4 & 2x^3 - 3x^2 \ -3x + 2 \\
 & 2x^3 \qquad\quad -8x \\
\hline
 & -3x^2 + 5x + 2 \\
 & -3x^2 \qquad\ +12 \\
\hline
 & 5x - 10
\end{array}
$$

$5x - 10$ ← Remainder

6. (i) Express 4^x in terms of y, where $y = 2^x$.

[1]

(ii) Hence find the values of x that satisfy the equation

$$3(4^x) - 10(2^x) + 3 = 0,$$

giving your answers correct to 2 decimal places.

[5]

(i) $4^x = (2^2)^x = 2^{2x} = (2^x)^2 = y^2$

(ii) Let $y = 2^x$

$$3y^2 - 10y + 3 = 0$$

$$(3y-1)(y-3) = 0$$

$y = \frac{1}{3}$ OR $y = 3$

$2^x = \frac{1}{3}$ OR $2^x = 3$

$\log_2 \frac{1}{3} = x$ OR $\log_2 3 = x$

$x = -1.58$ OR 1.58 (3SF)