



# REVISION PACK 1 AVERAGES, SPREAD & DIAGRAMS

- 1. A group of 10 married couples and 3 single men found that the mean age of the 10 women was 41.2 years and the standard deviation of the women's ages was 15.1 years. For the 13 men, the mean age  $\bar{x}_m$  was 46.3 years and the standard deviation was 12.7 years.
  - (i) Find the mean age of the whole group of 23 people.
  - (ii) The individual women's ages are denoted by  $x_w$  and the individual men's ages by  $x_m$ . By first finding  $\sum x_w^2$  and  $\sum x_m^2$ , find the standard deviation for the whole group.

[5]

[2]

2. Rachel measured the lengths in millimetres of some of the leaves on a tree. Her results are recorded below.

32 35 45 37 38 44 33 39 36 45

Find the mean and standard deviation of the lengths of these leaves.

[3]

**3.** A summary of 24 observations of *x* gave the following information:

$$\Sigma(x-a) = -73.2$$
 and  $\Sigma(x-a)^2 = 2115$ .

The mean of these values of x is 8.95.

- (i) Find the value of the constant *a*.
- (ii) Find the standard deviation of these values of *x*.

[2]

[2]

**4.** The length of time, *t* minutes, taken to do the crossword in a certain newspaper was observed on 12 occasions. The results are summarised below.

$$\Sigma(t-35) = -15$$
  $\Sigma(t-35)^2 = 82.23$ 

Calculate the mean and standard deviation of these times taken to do the crossword.

[4]

5. The following table shows the results of a survey to find the average daily time, in minutes, that a group of schoolchildren spent in internet chat rooms.

Time per day ( <i>t</i> minutes)	Frequency
$0 \le t < 10$	2
$10 \le t < 20$	f
$20 \le t < 40$	11
$40 \le t < 80$	4

The mean time was calculated to be 27.5 minutes.

(i) Form an equation involving f and hence show that the total number of children in the survey was 26.

[4]

(ii) Find the standard deviation of these times.

[2]

6. As part of a data collection exercise, members of a certain school year group were asked how long they spent on their Mathematics homework during one particular week. The times are given to the nearest 0.1 hour. The results are displayed in the following table.

Time spent ( <i>t</i> hours)	$0.1 \le t \le 0.5$	$0.6 \le t \le 1.0$	$1.1 \le t \le 2.0$	$2.1 \le t \le 3.0$	$3.1 \le t \le 4.5$
Frequency	11	15	18	30	21

(i) Draw, on graph paper, a histogram to illustrate this information.

[5]

(ii) Calculate an estimate of the mean time spent on their Mathematics homework by members of this year group.

[3]

7. The lengths of time in minutes to swim a certain distance by the members of a class of twelve 9-year-olds and by the members of a class of eight 16-year-olds are shown below.

9-year-olds: 13.0 16.1 16.0 14.4 15.9 15.1 14.2 13.7 16.7 16.4 15.0 13.2 16-year-olds: 14.8 13.0 11.4 11.7 16.5 13.7 12.8 12.9

- (i) Draw a back-to-back stem-and-leaf diagram to represent the information above.
- (ii) A new pupil joined the 16-year-old class and swam the distance. The mean time for the class of nine pupils was now 13.6 minutes. Find the new pupil's time to swim the distance.

[3]

[4]

8. Each father in a random sample of fathers was asked how old he was when his first child was born. The following histogram represents the information.



(iv) Find the probability that a father, chosen at random from the group, was between 25 and 30 years old when his first child was born, given that he was older than 25 years.

[2]

9. The weights of 30 children in a class, to the nearest kilogram, were as follows.

50	45	61	53	55	47	52	49	46	51
60	52	54	47	57	59	42	46	51	53
56	48	50	51	44	52	49	58	55	45

Construct a grouped frequency table for these data such that there are five equal class intervals with the first class having a lower boundary of 41.5 kg and the fifth class having an upper boundary of 61.5 kg.

[4]

**10.** A study of the ages of car drivers in a certain country produced the results shown in the table.

	Young	Middle-aged	Elderly
Males	40	35	25
Females	20	70	10

### Percentage of drivers in each age group

Illustrate these results diagrammatically.

[4]

**11.** The pulse rates, in beats per minute, of a random sample of 15 small animals are shown in the following table.

	-	115	120	158	132	125		
		104	142	160	145	104		
		162	117	109	124	134		
(i)	Draw a stem	-and-leaf di	agram to repr	resent the dat	ta.		[3]	
(ii)	Find the median and the quartiles.							
(iii)	i) On graph paper, using a scale of 2 cm to represent 10 beats per minute, draw a							
	UUX-allu-will	iskei piot oi	une uala.				[3]	

**12.** The following back-to-back stem-and-leaf diagram shows the cholesterol count for a group of 45 people who exercise daily and for another group of 63 who do not exercise. The figures in brackets show the number of people corresponding to each set of leaves.

	People who exercise		People who do not exercise		
(9)	987643221	3	1577	(4)	
(12)	988876653322	4	234458	(6)	
(9)	877765331	5	1 2 2 2 3 4 4 5 6 7 8 8 9	(13)	
(7)	6666432	6	1 2 3 3 3 4 5 5 5 7 7 8 9 9	(14)	
(3)	841	7	2 4 5 5 6 6 7 8 8	(9)	
(4)	9552	8	1 3 3 4 6 7 9 9 9	(9)	
(1)	4	9	1 4 5 5 8	(5)	
(0)		10	3 3 6	(3)	

Key: 2 | 8 | 1 represents a cholesterol count of 8.2 in the group who exercise and 8.1 in the group who do not exercise

(i) Give one useful feature of a stem-and-leaf diagram.

[1]

[3]

(ii) Find the median and the quartiles of the cholesterol count for the group who do not exercise.

You are given that the lower quartile, median and upper quartile of the cholesterol count for the group who exercise are 4.25, 5.3 and 6.6 respectively.

(iii) On a single diagram on graph paper, draw two box-and-whisker plots to illustrate the data.

**13.** During January the numbers of people entering a store during the first hour after opening were as follows.

Time after opening, <i>x</i> minutes	Frequency	Cumulative frequency
$0 < x \le 10$	210	210
$10 < x \le 20$	134	344
$20 < x \le 30$	78	422
$30 < x \le 40$	72	а
$40 < x \le 60$	b	540

- (i) Find the values of *a* and *b*.
- (ii) Draw a cumulative frequency graph to represent this information. Take a scale of 2 cm for 10 minutes on the horizontal axis and 2 cm for 50 people on the vertical axis.
- (iii) Use your graph to estimate the median time after opening that people entered the store.
- (iv) Calculate estimates of the mean, *m* minutes, and standard deviation, *s* minutes, of the time after opening that people entered the store.
- (v) Use your graph to estimate the number of people entering the store between  $\left(m \frac{1}{2}s\right)$

and  $\left(m + \frac{1}{2}s\right)$  minutes after opening.

[2]

## [2]

[4]

[2]

[4]

14. The arrival times of 204 trains were noted and the number of minutes, *t*, that each train was late was recorded. The results are summarised in the table.

Number of minutes late ( <i>t</i> )	$-2 \le t < 0$	$0 \le t < 2$	$2 \le t < 4$	$4 \le t < 6$	$6 \le t < 10$
Number of trains	43	51	69	22	19

- (i) Explain what  $-2 \le t < 0$  means about the arrival times of trains.
- (ii) Draw a cumulative frequency graph, and from it estimate the median and the interquartile range of the number of minutes late of these trains.

[7]

[1]

- **15.** In a survey, people were asked how long they took to travel to and from work, on average. The median time was 3 hours 36 minutes, the upper quartile was 4 hours 42 minutes and the interquartile range was 3 hours 48 minutes. The longest time taken was 5 hours 12 minutes and the shortest time was 30 minutes.
  - (i) Find the lower quartile.

[2]

[4]

(ii) Represent the information by a box-and-whisker plot, using a scale of 2 cm to represent 60 minutes.

16. The stem-and-leaf diagram below represents data collected for the number of hits on an internet site on each day in March 2007. There is one missing value, denoted by x.

	I									
0	0	1	5	6						(4)
1	1	3	5	6	6	8				(6)
2	1	1	2	3	4	4	4	8	9	(9)
3	1	2	2	2	x	8	9			(7)
4	2	5	6	7	9					(5)

Key: 1 5 represents 15 hits

- (i) Find the median and lower quartile for the number of hits each day [2]
- (ii) The interquartile range is 19. Find the value of *x*.

[2]

**17.** The salaries, in thousands of dollars, of 11 people, chosen at random in a certain office, were found to be:

40, 42, 45, 41, 352, 40, 50, 48, 51, 49, 47.

Choose and calculate an appropriate measure of central tendency (mean, mode or median) to summarise these salaries. Explain briefly why the other measures are not suitable.

[3]

# SOLUTIONS



	For multiplying by 10 and 13 respectively and dividing by 23		
	= 44.1 For correct answer		A12
	(ii) $15.1^2 = \frac{\sum x_w^2}{10} - 41.2^2$ For correct substitution from recognisable formula with or without sq rt		M1
	$\sum x_w^2 = 19254.5$ For correct $\sum x_w^2$ (can be rounded)		
	$12.7^2 = \frac{\sum x_m^2}{13} - 46.3^2$		A1
	$\sum x_m^2 = 29964.74$ For correct $\sum x_m^2$ (can be rounded)		A1
	Total $\Sigma = 49219.24$ For using 23 and their answer to (i) in correct formula		M1
	Sd = $\sqrt{\left(\frac{49219.24}{23} - 44.1^2\right)} = 14.0$ For correct answer		A15
2.	mean = 38.4 mm Correct answer		[7] B1
	Correct method if shown (can be implied) must see a $$ sign		M1
	sd = 4.57 mm c.a.o Correct answer		A1 [ <b>3</b> ]
3.	(i) $-73.2/24 (= -3.05)$ Accept (-72.4 + anything)/ 24	M1	
	a = 8.95 + 3.05 = 12 Correct answer		A1
	OR		
	$8.95 \times 24 \ (= 214.8)$ $\Sigma x - \Sigma a = -73.2$ For $8.95 \times 24$ seen		M1
	$\Sigma a = 288 \ a = 12$		A12

M1

 $(41.2 \times 10 + 46.3 \times 13) / 23$ 

1.

(i)

Correct answer obtained using  $\Sigma x$  and  $\Sigma a$ 

(ii) standard deviation = 
$$\sqrt{\frac{2115}{24} - (-3.05)^2}$$
 M1  
For  $\frac{2115}{24} - (\pm \text{ their coded mean})^2$   
= 8.88  
Correct answer

OR

 $sd = \sqrt{\frac{3814.2}{24} - 8.95^2}$  M1

For 
$$\frac{\text{their } \sum x^2}{24} - 8.95^2$$
 where  $\Sigma x^2$  is obtained  
from expanding  $\Sigma (x - a)^2$  with  $2a\Sigma x$  seen

4. mean = 35 - 15/12 M1 For -15/12 seen

$$sd = \sqrt{82.23/12 - (-15/12)^2}$$
 M1

 $82.23/12 - (\pm \text{their coded mean})^2$ 

5. (i)  $5 \times 2 + 15f + 30 \times 11 + 60 \times 4 = 27.5(17 + f)$ For attempt at LHS, accept end points or cl width

For attempt at RHS, must have 17+f

$$f = 9$$
  
For correct  $f$ 

total = 26 AG For correct answer given, ft if previous answer rounds to 9

(ii) 
$$\sigma = 16.1$$
 M1  
For method including sq rt and mean  
squared (can be implied if using calculator,

[4]

[4]

A1

M1

A1

A14

**M**1

must be  $x^2 f$  on mid-points) or  $\sum \frac{f(x-\bar{x})^2}{26}$ 

For correct answer

A12

**B**1

**B**1

B15

M1

M1

A13

M1

[6]

6.

(i)

Fd: 22, 30, 18, 30, 14 Attempt at freq density or scaling



Bar lines correctly located at 0.55, 1.05, 2.05, 3.05, no gaps

correct widths of bars

both axes uniform from at least 0 to 15 or 30, and 0.05 to 4.5 and labelled, (fd, or freq per half hour, time, hours, t)

(ii) mid-points 0.3, 0.8, 1.55, 2.55, 3.8 an attempt at mid-points (not class widths)

> = 199.5 / 95using ( $\Sigma$  their fx) / their 95

mean = 2.1 hours correct answer from 199.5 in num

### **7.** (i)

16 yr olds		9 year olds
7, 4	11	
9, 8,	12	
7,0	13	0, 2, 7,
8	14	2, 4,

[8]

		15	0 1 9				
	5	16	0, 1, 4, 7,	-			
	3 columns including stem in the middle, st in leaves. Can go dow	J		B1			
	One leaf column corr not necessary	rect, orderi	ng			B1	
	Other leaf column co (ordering not nec) an leaves labelled correct be in key)	orrect d both ctly (could				B1	
	Key 7  13  2 means 13 minutes Key correct both way keys one each way, n minutes	3.7 minutes ys or two nust have	s and 13.2			B14	
(ii)	$\sum_{106.8 \text{ seen or implied}} (8 \text{ pupils}) = 10$	06.8 1				B1	
	$\sum_{\text{for } 13.6 \times 9} (9 \text{ pupils}) = 13$	3.6 × 9 (= )	122.4)			B1	
	New pupil's time = 1 Ft on $122.4 - $ their	$5.6 \min \sum_{k=1}^{\infty} 8^{k}$				B1ft3	[7]
(i)	30-35 years				B1	1	[,]
(ii)	$4.8 \times 5$ Multiplying by 5					M1	
	= 24 Correct answer					A12	
(iii)	4 + 18 + 24 + 28 + 20 Summing their 6 atte	6 +10 mpts at fre	equencies			M1	
	= 110 Correct answer					A12	
(iv)	24 /88 Dividing their (ii) by	their atten	npt at > 25 group			M1	
	= 0.273 Correct answer, ft on	above				A1 ft2	[7]

8.

		Weight	freq		
		41.5-45.5	4		
		45.5-49.5	7		
		49.5-53.5	10		
		53.5-57.5	5		
		57.5-61.5	4		
	Five groups			M1 A1	l
	Attempt to calculate frequen	M	l		
	5 frequencies correct			Al	[4]
10.	two pie charts or 2 bars (m a 3 lots of 2 or 2 lots of 3, bars	nd f) s, lines or sectors		M	L
	3 different age categories in group one category touching, not s category not touching, bars e	Al	l		
	correct height or angle accept pie chart visually corr	B	l		
	labels m and f, percentage, d elderly	B	[4]		
1.	(i)				
	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$				
	Correct stem			B	l
	Correct leaves, must b and give correct overa	e sorted and in column ll shape	s	BI	

11.

Key, must have vertical line in both

(ii) median = 125Any 2 correct values seen

> LQ = 115 UO = 145 third correct value

(iii)



correct uniform scale from at least 110 to 160 with room for end points, and label or title

correct median and quartiles on diagram ft their values (must be box ends)

correct whiskers, no line through box, touching box in the middle not the top or bottom

12. (i) shows all the data Or other suitable advantage e.g. can see the shape, mode etc.

B12

B1

B1ft

B13

**B**1

1

**B**1

[8]

(ii)	Not exercise $LQ = 5.4$	B1
	Median = 6.5	B1ft
	UQ = 8.3	B1ft3
	ft on first answer missing the decimal point	





*b* = 46

(ii)



B1

B12

[8]

		B1 Correct linear se	cale mi	nimum	0 to 54	0 and 0	) to 60					
		Labels (cf or pe or minutes) and	ople or attemp	numbe t at cf o	er of peo or cf ste	ople) ar p polyg	nd (time gon	,				B1
		Attempt to plot (30, 422), (40, 4	points a 194)	at (10, 1	210), (2	20, 344)	),					M1
		Correct graph th	rough	(0, 0) a	nd (60,	540)						A14
	(iii)	median is										M1
		Attempt to read Correct answer	from g	raph at	line y =	= 270 o	r 270.5	13.5 to 14.6 m	in			A12
	(iv)	$(5 \times 210 + 15 \times 35 \times 72 + 50 \times 4)$ = 9830 / 540	134 + 1 46) / 54	25 × 78 0	3 +							M1
		Using mid point = 18.2 min Correct mean	ts and f	requen	cies							A1
		$(5^2 \times 210 + 15^2 \times 134 +) - 18.2^2$ Attempt at $\Sigma x^2 f / \Sigma f$ – their mean <sup>2</sup> numerically, could use cfs, ucb, but not class widths								M1		
		sd = 14.2 min Correct answer										A14
	(v)	$18.2 \pm 7.1 = 11.$ 390 - 225	1, 25.3									M1
	Attempt to read their mean $\pm \frac{1}{2}$ sd from cf graph											
		= 155 to 170 pe Correct answer	ople									A12
14.	(i)	some trains were up to 2 minutes early B1 Or sensible equivalent, must use the idea 'early' 2 not needed cf table NB All M marks are independent					B1	1	[14]			
	(ii)											
		Min Late, Less than	0	2	4	6	10					

C freq



(ii)

15.



16.

17.

		Correct whiskers (accept hour decimals or minutes)		B1	
		Correct median line, can be broken or extended		B1	
		Correct UQ and LQ ft on their (i), box ends		B1ft	
		correct uniform scale label hours or minutes, could be heading or key		B14	
		th			[6]
	(i)	$median = 16^{m} along = 24$	B1		
		LQ = 16  not  15.5		B12	
	(ii)	UQ = LQ + 19 = 35 For adding 19 to their LQ in whatever form		M1	
		x = 5 Must be 5 not 35. c.w.o.		A12	[4]
					[י]
	medi	an		B1	
\$47000 Must have 47000			B1		
	data l Acce	have an outlier, are skew etc pt any equivalent reason		B1	
					[3]