

1. A group of 10 married couples and 3 single men found that the mean age of the 10 women was 41.2 years and the standard deviation of the women's ages was 15.1 years. For the 13 men, the mean age $\bar{x}_{m}$ was 46.3 years and the standard deviation was 12.7 years.
(i) Find the mean age of the whole group of 23 people.
(ii) The individual women's ages are denoted by $x_{w}$ and the individual men's ages by $x_{m}$. By first finding $\sum x_{w}^{2}$ and $\sum x_{m}^{2}$, find the standard deviation for the whole group.
2. Rachel measured the lengths in millimetres of some of the leaves on a tree. Her results are recorded below.

| 32 | 35 | 45 | 37 | 38 | 44 | 33 | 39 | 36 | 45 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Find the mean and standard deviation of the lengths of these leaves.
3. A summary of 24 observations of $x$ gave the following information:

$$
\Sigma(x-a)=-73.2 \quad \text { and } \quad \Sigma(x-a)^{2}=2115
$$

The mean of these values of $x$ is 8.95 .
(i) Find the value of the constant $a$.
(ii) Find the standard deviation of these values of $x$.
4. The length of time, $t$ minutes, taken to do the crossword in a certain newspaper was observed on 12 occasions. The results are summarised below.

$$
\Sigma(t-35)=-15 \quad \Sigma(t-35)^{2}=82.23
$$

Calculate the mean and standard deviation of these times taken to do the crossword.
5. The following table shows the results of a survey to find the average daily time, in minutes, that a group of schoolchildren spent in internet chat rooms.

| Time per day <br> $(t$ minutes $)$ | Frequency |
| :---: | :---: |
| $0 \leq t<10$ | 2 |
| $10 \leq t<20$ | $f$ |
| $20 \leq t<40$ | 11 |
| $40 \leq t<80$ | 4 |

The mean time was calculated to be 27.5 minutes.
(i) Form an equation involving $f$ and hence show that the total number of children in the survey was 26 .
(ii) Find the standard deviation of these times.
6. As part of a data collection exercise, members of a certain school year group were asked how long they spent on their Mathematics homework during one particular week. The times are given to the nearest 0.1 hour. The results are displayed in the following table.

| Time spent $(t$ hours $)$ | $0.1 \leq t \leq 0.5$ | $0.6 \leq t \leq 1.0$ | $1.1 \leq t \leq 2.0$ | $2.1 \leq t \leq 3.0$ | $3.1 \leq t \leq 4.5$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Frequency | 11 | 15 | 18 | 30 | 21 |

(i) Draw, on graph paper, a histogram to illustrate this information.
(ii) Calculate an estimate of the mean time spent on their Mathematics homework by members of this year group.
7. The lengths of time in minutes to swim a certain distance by the members of a class of twelve 9 -year-olds and by the members of a class of eight 16 -year-olds are shown below.

| 9-year-olds: |  | 13.0 | 16.1 | 16.0 | 14.4 | 15.9 | 15.1 | 14.2 | 13.7 | 16.7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16.4 | 15.0 | 13.2 |  |  |  |  |  |  |  |  |
| 16-year-olds: |  | 14.8 | 13.0 | 11.4 | 11.7 | 16.5 | 13.7 | 12.8 | 12.9 |  |

(i) Draw a back-to-back stem-and-leaf diagram to represent the information above.
(ii) A new pupil joined the 16-year-old class and swam the distance. The mean time for the class of nine pupils was now 13.6 minutes. Find the new pupil's time to swim the distance.
8. Each father in a random sample of fathers was asked how old he was when his first child was born. The following histogram represents the information.

(i) What is the modal age group?
(ii) How many fathers were between 25 and 30 years old when their first child was born?
(iii) How many fathers were in the sample?
(iv) Find the probability that a father, chosen at random from the group, was between 25 and 30 years old when his first child was born, given that he was older than 25 years.
9. The weights of 30 children in a class, to the nearest kilogram, were as follows.

| 50 | 45 | 61 | 53 | 55 | 47 | 52 | 49 | 46 | 51 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 60 | 52 | 54 | 47 | 57 | 59 | 42 | 46 | 51 | 53 |
| 56 | 48 | 50 | 51 | 44 | 52 | 49 | 58 | 55 | 45 |

Construct a grouped frequency table for these data such that there are five equal class intervals with the first class having a lower boundary of 41.5 kg and the fifth class having an upper boundary of 61.5 kg .
10. A study of the ages of car drivers in a certain country produced the results shown in the table.

Percentage of drivers in each age group

|  | Young | Middle-aged | Elderly |
| :--- | :---: | :---: | :---: |
| Males | 40 | 35 | 25 |
| Females | 20 | 70 | 10 |

Illustrate these results diagrammatically.
11. The pulse rates, in beats per minute, of a random sample of 15 small animals are shown in the following table.

| 115 | 120 | 158 | 132 | 125 |
| :--- | :--- | :--- | :--- | :--- |
| 104 | 142 | 160 | 145 | 104 |
| 162 | 117 | 109 | 124 | 134 |

(i) Draw a stem-and-leaf diagram to represent the data.
(ii) Find the median and the quartiles.
(iii) On graph paper, using a scale of 2 cm to represent 10 beats per minute, draw a box-and-whisker plot of the data.
12. The following back-to-back stem-and-leaf diagram shows the cholesterol count for a group of 45 people who exercise daily and for another group of 63 who do not exercise. The figures in brackets show the number of people corresponding to each set of leaves.

People who exercise People who do not exercise

| 987643221 | 3 | 1577 | (4) |
| :---: | :---: | :---: | :---: |
| 988876653322 | 4 | 234458 | (6) |
| 877765331 | 5 | 1222344567889 | (13) |
| 6666432 | 6 | 12333455577899 | (14) |
| 841 | 7 | 245566788 | (9) |
| 9552 | 8 | 133467999 | (9) |
| 4 | 9 | 14558 | (5) |
|  | 10 | 336 | (3) |

Key: $2|8| 1$ represents a cholesterol count of 8.2 in the group who exercise and 8.1 in the group who do not exercise
(i) Give one useful feature of a stem-and-leaf diagram.
(ii) Find the median and the quartiles of the cholesterol count for the group who do not exercise.

You are given that the lower quartile, median and upper quartile of the cholesterol count for the group who exercise are $4.25,5.3$ and 6.6 respectively.
(iii) On a single diagram on graph paper, draw two box-and-whisker plots to illustrate the data.
13. During January the numbers of people entering a store during the first hour after opening were as follows.

| Time after opening, <br> $x$ minutes | Frequency | Cumulative <br> frequency |
| :---: | :---: | :---: |
| $0<x \leq 10$ | 210 | 210 |
| $10<x \leq 20$ | 134 | 344 |
| $20<x \leq 30$ | 78 | 422 |
| $30<x \leq 40$ | 72 | $a$ |
| $40<x \leq 60$ | $b$ | 540 |

(i) Find the values of $a$ and $b$.
(ii) Draw a cumulative frequency graph to represent this information. Take a scale of 2 cm for 10 minutes on the horizontal axis and 2 cm for 50 people on the vertical axis.
(iii) Use your graph to estimate the median time after opening that people entered the store.
(iv) Calculate estimates of the mean, $m$ minutes, and standard deviation, $s$ minutes, of the time after opening that people entered the store.
(v) Use your graph to estimate the number of people entering the store between $\left(m-\frac{1}{2} s\right)$ and $\left(m+\frac{1}{2} s\right)$ minutes after opening.
14. The arrival times of 204 trains were noted and the number of minutes, $t$, that each train was late was recorded. The results are summarised in the table.

| Number of minutes late $(t)$ | $-2 \leq t<0$ | $0 \leq t<2$ | $2 \leq t<4$ | $4 \leq t<6$ | $6 \leq t<10$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Number of trains | 43 | 51 | 69 | 22 | 19 |

(i) Explain what $-2 \leq t<0$ means about the arrival times of trains.
(ii) Draw a cumulative frequency graph, and from it estimate the median and the interquartile range of the number of minutes late of these trains.
15. In a survey, people were asked how long they took to travel to and from work, on average. The median time was 3 hours 36 minutes, the upper quartile was 4 hours 42 minutes and the interquartile range was 3 hours 48 minutes. The longest time taken was 5 hours 12 minutes and the shortest time was 30 minutes.
(i) Find the lower quartile.
(ii) Represent the information by a box-and-whisker plot, using a scale of 2 cm to represent 60 minutes.
16. The stem-and-leaf diagram below represents data collected for the number of hits on an internet site on each day in March 2007. There is one missing value, denoted by $x$.

| 0 | 0 | 1 | 5 | 6 |  |  |  |  |  | $(4)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 3 | 5 | 6 | 6 | 8 |  |  |  | $(6)$ |
| 2 | 1 | 1 | 2 | 3 | 4 | 4 | 4 | 8 | 9 | $(9)$ |
| 3 | 1 | 2 | 2 | 2 | $x$ | 8 | 9 |  |  | $(7)$ |
| 4 | 2 | 5 | 6 | 7 | 9 |  |  |  |  | $(5)$ |

Key: $1 \mid 5$ represents 15 hits
(i) Find the median and lower quartile for the number of hits each day
(ii) The interquartile range is 19 . Find the value of $x$.
17. The salaries, in thousands of dollars, of 11 people, chosen at random in a certain office, were found to be:
$40, \quad 42,45, \quad 41, \quad 352,40, \quad 50,48, \quad 51,49,47$.
Choose and calculate an appropriate measure of central tendency (mean, mode or median) to summarise these salaries. Explain briefly why the other measures are not suitable.

## SOLUTIONS



1. (i) $(41.2 \times 10+46.3 \times 13) / 23$

For multiplying by 10 and 13 respctively and dividing by 23
$=44.1$
For correct answer
(ii) $15.1^{2}=\frac{\sum x_{w}{ }^{2}}{10}-41.2^{2}$

For correct substitution from recognisable formula with or without sq rt
$\sum x_{w}{ }^{2}=19254.5$
For correct $\sum x_{w}{ }^{2}$ (can be rounded)
$12.7^{2}=\frac{\sum x_{m}{ }^{2}}{13}-46.3^{2}$
$\sum x_{m}{ }^{2}=29964.74$
For correct $\sum x_{m}{ }^{2}$ (can be rounded)

Total $\Sigma=49219.24$
For using 23 and their answer to (i) in correct formula
$\mathrm{Sd}=\sqrt{\left(\frac{49219.24}{23}-44.1^{2}\right)}=14.0$
For correct answer
2. mean $=38.4 \mathrm{~mm}$

Correct answer

Correct method if shown (can be implied) must
see a $\sqrt{ }$ sign
$\mathrm{sd}=4.57 \mathrm{~mm} \mathrm{c.a.o}$
Correct answer
3. (i) $-73.2 / 24(=-3.05)$

Accept $(-72.4+$ anything $) / 24$
$a=8.95+3.05=12$
Correct answer

OR

| $8.95 \times 24(=214.8)$ | M1 |
| :--- | :--- |
| $\Sigma x-\Sigma a=-73.2$ |  |
| For $8.95 \times 24$ seen |  |

$$
\begin{equation*}
\Sigma a=288 a=12 \tag{A12}
\end{equation*}
$$

Correct answer obtained using $\Sigma x$ and $\Sigma a$
(ii) standard deviation $=\sqrt{\frac{2115}{24}-(-3.05)^{2}}$

For $\frac{2115}{24}-( \pm \text { their coded mean })^{2}$
$=8.88$
Correct answer
OR
$\mathrm{sd}=\sqrt{\frac{3814.2}{24}-8.95^{2}}$
For $\frac{\text { their } \sum x^{2}}{24}-8.95^{2}$ where $\Sigma \mathrm{x}^{2}$ is obtained
from expanding $\Sigma(x-a)^{2}$ with $2 a \Sigma x$ seen
$=8.88$
Correct answer
4. mean $=35-15 / 12$

For $-15 / 12$ seen
$=33.75$ (33.8) minutes
A1
Correct answer
$s d=\sqrt{82.23 / 12-(-15 / 12)^{2}}$
$82.23 / 12-( \pm \text { their coded mean })^{2}$
$=2.3$ minutes
A1
Correct answer
5. (i) $5 \times 2+15 f+30 \times 11+60 \times 4=27.5(17+f)$

M1
For attempt at LHS, accept end points or cl width

For attempt at RHS, must have $17+f$
$f=9$
For correct $f$
total $=26 \mathrm{AG}$
For correct answer given, ft if previous answer rounds to 9
(ii) $\sigma=16.1$

For method including sq rt and mean squared (can be implied if using calculator,
must be $x^{2} f$ on mid-points) or $\sum \frac{f(x-\bar{x})^{2}}{26}$

For correct answer
6. (i) Fd: 22, 30, 18, 30, 14

Attempt at freq density or scaling


Bar lines correctly located at $0.55,1.05$, $2.05,3.05$, no gaps
correct widths of bars
both axes uniform from at least 0 to 15 or 30 , and 0.05 to 4.5 and labelled, (fd, or freq per half hour, time, hours, $t$ )
(ii) mid-points $0.3,0.8,1.55,2.55,3.8$ an attempt at mid-points (not class widths)
$=199.5 / 95$
using ( $\Sigma$ their $\mathrm{f} x$ ) / their 95
mean $=2.1$ hours
correct answer from 199.5 in num
7. (i)

| 16 yr olds |  | 9 year olds |
| :---: | :--- | :--- |
| 7,4 | 11 |  |
| 9,8, | 12 |  |
| 7,0 | 13 | $0,2,7$, |
| 8 | 14 | 2,4, |


|  | 15 | $0,1,9$, |
| :--- | :--- | :--- |
| 5 | 16 | $0,1,4,7$, |

3 columns including an integer
stem in the middle, single digits
in leaves. Can go downwards
One leaf column correct, ordering
not necessary

Other leaf column correct
(ordering not nec) and both
leaves labelled correctly (could be in key)

Key $7|13| 2$ means 13.7 minutes and 13.2
minutes
Key correct both ways or two
keys one each way, must have
minutes
(ii) $\quad \sum(8$ pupils $)=106.8$
106.8 seen or implied

$$
\sum(9 \text { pupils })=13.6 \times 9(=122.4)
$$

for $13.6 \times 9$
New pupil's time $=15.6 \mathrm{~min}$
Ft on 122.4 - their $\sum 8$
8. (i) 30-35 years

B1 1
$\begin{array}{ll}\text { (ii) } & 4.8 \times 5 \\ & \text { Multiplying by } 5\end{array}$
$=24$
Correct answer
$\begin{array}{ll}\text { (iii) } & 4+18+24+28+26+10 \\ \text { Summing their } 6 \text { attempts at frequencies } & \text { M1 }\end{array}$
$=110$
Correct answer
(iv) $24 / 88$

Dividing their (ii) by their attempt at $>25$ group
$=0.273$
A1 ft2
Correct answer, ft on above
9.

| Weight | freq |
| :---: | :---: |
| $41.5-45.5$ | 4 |
| $45.5-49.5$ | 7 |
| $49.5-53.5$ | 10 |
| $53.5-57.5$ | 5 |
| $57.5-61.5$ | 4 |

Five groups

Correct boundaries, accept 42-45, 46-49 etc

Attempt to calculate frequencies $\Sigma 29,30$ or 31 .

5 frequencies correct
10. two pie charts or 2 bars ( $m$ and $f$ )

3 lots of 2 or 2 lots of 3 , bars, lines or sectors

3 different age categories in each
group
one category touching, not superimposed, one
category not touching, bars equal width
correct height or angle
accept pie chart visually correct
labels m and f , percentage, drivers, $\mathrm{y}, \mathrm{m}$
elderly
11. (i)

|  |  |  |
| :--- | :--- | :--- |
| 10 | 4 | 4 |
| 11 | 5 | 7 |
| 12 | 0 | 4 |
| 13 | 2 | 4 |
| 14 | 2 | 5 |
| 15 | 8 |  |
| 16 | 0 | 8 |

Correct stem

Correct leaves, must be sorted and in columns and give correct overall shape
key 4 represents 104

Key, must have vertical line in both
(ii) median $=125$

Any 2 correct values seen
$L Q=115$
$\mathrm{UQ}=145$
third correct value
(iii)

correct uniform scale from at least 110 to 160 with room for end points, and label or title
correct median and quartiles on diagram ft their values (must be box ends)
correct whiskers, no line through box, touching box in the middle not the top or bottom
12. (i) shows all the data

Or other suitable advantage e.g. can see the shape, mode etc.
(ii) Not exercise LQ = 5.4

Median $=6.5$

$$
\mathrm{UQ}=8.3
$$

ft on first answer missing the decimal point
(iii)


For one linear numbered scale from 3 to 9.5 , or two identically positioned scales

For not exercise all correct on linear scale

For exercise correct on linear scale

For two labels and cholesterol and scale labelled

SR non linear scale max B0 B0 B0 B1
SR no graph paper lose one mark
13. (i) $a=494$

B1
$b=46$
B12
(ii)


B1
Correct linear scale minimum 0 to 540 and 0 to 60

Labels (cf or people or number of people) and (time, or minutes) and attempt at cf or cf step polygon

Attempt to plot points at $(10,210),(20,344)$, $(30,422),(40,494)$

Correct graph through $(0,0)$ and $(60,540)$
(iii) median is

Attempt to read from graph at line $y=270$ or 270.513 .5 to 14.6 min Correct answer
(iv) $(5 \times 210+15 \times 134+25 \times 78+$
$35 \times 72+50 \times 46) / 540$
$=9830 / 540$
Using mid points and frequencies
$=18.2 \mathrm{~min}$
Correct mean
$\left(5^{2} \times 210+15^{2} \times 134+\ldots.\right)-18.2^{2}$
Attempt at $\Sigma x^{2} f / \Sigma f-$ their mean ${ }^{2}$ numerically,
could use cfs, ucb, but not class widths
$\mathrm{sd}=14.2 \mathrm{~min}$
Correct answer
(v) $18.2 \pm 7.1=11.1,25.3$

390-225
Attempt to read their mean $\pm 1 / 2$ sd from cf graph
$=155$ to 170 people
Correct answer
14. (i) some trains were up to 2 minutes early

Or sensible equivalent, must use the idea 'early'
2 not needed
(ii) cf table

NB All M marks are independent

| Min Late, <br> Less than | 0 | 2 | 4 | 6 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| C freq | 43 | 94 | 163 | 185 | 204 |



Attempt at C F table with upper limits no halves

Uniform linear scales from at least 0 to 10 and 0 to 204 and at least one axis labelled, CF or mins or t

Attempt at graph their 5 points. $(-2,0)$ not nec (could be midpoints or lower bounds not f d )

Attempt at median along 102 or 102.5 line

Attempt at LQ along 51/52 line and UQ along 153/154 line from graph

Median = rounding to 2.1 to 2.4 min
Correct median
IQ range $=$ rounding to 3.2 to 3.6 min
Correct IQ range allow from midpoints etc
15. (i) $\mathrm{LQ}=4 \mathrm{hr} 42 \mathrm{~min}-3 \mathrm{hr} 48 \mathrm{~min}$

Subtracting IQR from UQ

$$
=54 \mathrm{~min} \text { (0.9 hours) }
$$

Correct answer
(ii)


Correct whiskers (accept hour decimals or minutes)

Correct median line, can be broken or extended

Correct UQ and LQ ft on their (i), box ends
correct uniform scale label hours or minutes, could be heading or key
16. (i) median $=16^{\text {th }}$ along $=24$
$\begin{array}{ll}\mathrm{LQ}=16 \text { not } 15.5 & \text { B12 }\end{array}$
(ii) $\mathrm{UQ}=\mathrm{LQ}+19=35$

For adding 19 to their LQ in whatever form
$x=5$
Must be 5 not 35. c.w.o.
17. median B1
\$47000
B1
Must have 47000
data have an outlier, are skew etc B1
Accept any equivalent reason

