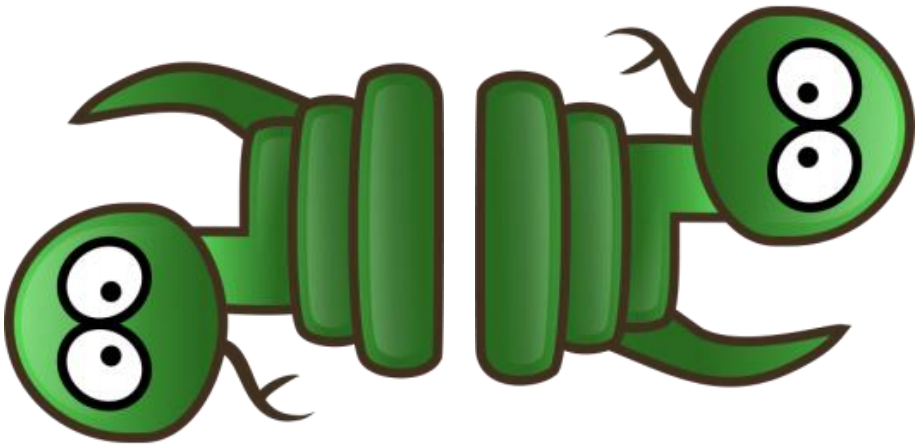
# S1



# REVISION PACK 2

## PERMUTATIONS & COMBINATIONS

**1.** A choir consists of 13 sopranos, 12 altos, 6 tenors and 7 basses. A group consisting of 10 sopranos, 9 altos, 4 tenors and 4 basses is to be chosen from the choir.

(i) In how many different ways can the group be chosen?

**[2]**

(ii) In how many ways can the 10 chosen sopranos be arranged in a line if the 6 tallest stand next to each other?

**[3]**

(iii) The 4 tenors and 4 basses in the group stand in a single line with all the tenors next to each other and all the basses next to each other. How many possible arrangements are there if three of the tenors refuse to stand next to any of the basses?

**[3]**

**2.** (i) Find the number of ways in which all twelve letters of the word REFRIGERATOR can be arranged

(a) if there are no restrictions,

**[2]**

(b) if the Rs must all be together.

**[2]**

(ii) How many different selections of four letters from the twelve letters of the word REFRIGERATOR contain no Rs and two Es?

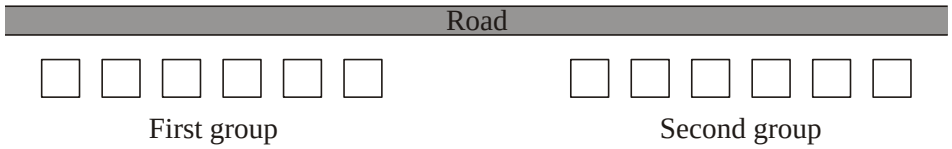
**[3]**

**3.** A builder is planning to build 12 houses along one side of a road. He will build 2 houses in style *A*, 2 houses in style *B*, 3 houses in style *C*, 4 houses in style *D* and 1 house in style *E*.

(i) Find the number of possible arrangements of these 12 houses.

**[2]**

(ii)




The 12 houses will be in two groups of 6 (see diagram). Find the number of possible arrangements if all the houses in styles *A* and *D* are in the first group and all the houses in styles *B*, *C* and *E* are in the second group.

**[3]**

(iii) Four of the 12 houses will be selected for a survey. Exactly one house must be in style *B* and exactly one house in style *C*. Find the number of ways in which these four houses can be selected.

**[2]**

**4.** Issam has 11 different CDs, of which 6 are pop music, 3 are jazz and 2 are classical.

(i) How many different arrangements of all 11 CDs on a shelf are there if the jazz CDs are all next to each other?

**[3]**

(ii) Issam makes a selection of 2 pop music CDs, 2 jazz CDs and 1 classic CD.
How many different possible selections can be made?

**[3]**

**5.** The six digits 4, 5, 6, 7, 7, 7 can be arranged to give many different 6-digit numbers.

(i) How many different 6-digit numbers can be made?

**[2]**

(ii) How many of these 6-digit numbers start with an odd digit and end with an odd digit?

**[4]**

**6.** Six men and three women are standing in a supermarket queue.

(i) How many possible arrangements are there if there are no restrictions on order?

**[2]**

(ii) How many possible arrangements are there if no two of the women are standing next to each other?

**[4]**

(iii) Three of the people in the queue are chosen to take part in a customer survey. How many different choices are possible if at least one woman must be included?

**[3]**

and *E* are in the second group.

**[3]**

(iii) Four of the 12 houses will be selected for a survey. Exactly one house must be in style *B* and exactly one house in style *C*. Find the number of ways in which these four houses can be selected.

**[2]**

**7.** (a) A football team consists of 3 players who play in a defence position, 3 players who play in a midfield position and 5 players who play in a forward position. Three players are chosen to collect a gold medal for the team. Find in how many ways this can be done

(i) if the captain, who is a midfield player, must be included, together with one defence and one forward player,

**[2]**

(ii) if exactly one forward player must be included, together with any two others.

**[2]**

(b) Find how many different arrangements there are of the nine letters in the words **GOLD MEDAL**

(i) if there are no restrictions on the order of the letters,

**[2]**

(ii) if the two letters D come first and the two letters L come last.

**[2]**

**8.** A staff car park at a school has 13 parking spaces in a row. There are 9 cars to be parked.

(i) How many different arrangements are there for parking the 9 cars and leaving 4 empty spaces?

**[2]**

(ii) How many different arrangements are there if the 4 empty spaces are next to each other?

**[3]**

(iii) If the parking is random, find the probability that there will **not** be 4 empty spaces next to each other.

**[2]**

# SOLUTIONS

**1.** (i) $^{13}C_{10} \times {}^{12}C_9 \times {}^{6}C_4 \times {}^{7}C_4$ M1
Expression involving the product of 4 combinations

= 33033000 (33000000) A12
Correct final answer allow $33\times10^6$ or $3.3\times10^7$

(ii) $5! \times 6!$ B1
6! or 5! or 4! oe seen no denom

= 86400 M1
a single product involving 6! and either 4! or 5!
no denom

A13
Correct final answer

(iii) $4! \times 3! \times 2$ B1
4! or 3! or 4!/4 seen

M1
a single product involving 3! (or 4!/4) and 4!

= 288 A13
Correct final answer

**[8]**

**2.** (i) (a) $\frac{12!}{4!2!}$ = 9979200 (9980000) B1
Dividing by 4! and 2! only

Correct answer B12

(b) $\frac{9!}{2!}$ = 181440 (181000) B1
9! or $9 \times 8!$ seen not in denom

Correct answer B12

(ii) ${}_6C_2$ or ${}_4C_0 \times {}_2C_2 \times {}_6C_2$
or ${}_6C_4$ or ${}_6P_2/2!$ M1
for seeing ${}_6C_{\text{something}}$ or ${}_6P_{\text{something}}$
in a product (could be with 1)

M1

for seeing ${}_{\text{something}}C_2$ or ${}_6C_4$

= 15 A13
correct answer

15 with no working scores full
marks

**[7]**

**3.** (i) $\frac{12!}{2!2!3!4!} = 831600$ M1
Dividing by 3! 4! and 2! once or twice o.e

A12

Correct final answer

(ii) $\frac{6!}{4!2!} \times \frac{6!}{2!3!}$ B1
$\frac{6!}{4!2!}$ and $\frac{6!}{2!3!}$ seen o.e

M1

multiplying their numbers for group 1 with their
numbers for group 2

= 900 A13
correct final answer

(iii) $2 \times 3 \times {}_7C_2$ or $2 \times 3 \times 21$ M1
${}_7C_2$ seen multiplied or 5 options added

= 126 A12
correct final answer

**[7]**

**4.** (i) $3! \times 8! \times 9$ M1

For $k3!$ seen, $k$ a + ve integer, accept ${}_3P_3$

M1

For using $m8!$ or $n9!$ seen, $m$ and $n$ + ve integers, accept $m\ {}_8P_8$ etc

= 2,177,280 or 2,180,000 A13

Correct final answer

(ii) ${}_6C_2 \times {}_3C_2 \times {}_2C_1$ M1

Multiplying 3 combinations or 3 numbers or 3 permutations together only

B1

All of ${}_6C_2$ and ${}_3C_2$ and ${}_2C_1$ seen

(15, 3, 2)

= 90 A13

Correct answer

**[6]**

**5.** (i) $\frac{6!}{3!} = 120$ M1

For dividing by 3!

Correct answer

A12

(ii) $5 \ldots . 7 = \frac{4!}{2!} = 12$ M1

For identifying different cases

$7 \ldots . 5 = \frac{4!}{2!} = 12$ B1

For 4!/2! seen

$7 \ldots . 7 = 4! = 24$ B1

For 4! alone seen or in a sum or product

total = 48 A14

Correct final answer

**[6]**

**6.** (i) 9! B1

9! Or ${}_9P_9$ only

= 362880 (363000) B12

correct answer

(ii) $6! \times {}_7P_3$ B1
6! Seen

M1

${}_7P$ or ${}_7C_{\text{something}}$ or 7 multiplied by something

=151200 A1
mult by ${}_7P_3$

A14

correct answer

(iii) 1 woman: ${}_3C_1 \times {}_6C_2 = 45$ M1
summing cases for 1, 2, 3 women

2 women: ${}_3C_2 \times {}_6C_1 = 18$ B1
one correct case

3 women: ${}_3C_3$ =1
total = 64 A1
correct answer

**OR:**

no restrictions ${}_9C^3$ (84) B1
${}_9C_3$ or 84 or 3 times ${}_8C_2$ seen

Men only M1
attempt at subt of their 'no women' case

84 – 20 = 64 A13
correct answer

**[9]**

**7.** (a) (i) ${}_3C_1 \times {}_5C_1$ M1
For multiplying two combinations together
For correct answer = 15

B12

(ii) ${}_5C_1 \times {}_6C_2$ M1
For seeing ${}_6C_2$, or separating it into three alternatives either added or multiplied

= 75 A12
For correct answer

(b) (i) 9!/2!2! = 90720 M1

For dividing by 2! twice

A12

For correct answer

(ii) 5! Or $_5P_5$ B1
5! seen in a numerator

= 120 B12
For correct final answer

**[8]**

**8.** (i) $_{13}P_9$ = 259,459,200 or 259,000,000 M1
For using a permutation involving 13

A12

For correct answer

(ii) 10! or $_{10}P_9$ = 3628800 M1
For using a 10

M1

For using a 9!

A13

For correct answer

(iii) 1 – (ii) / (i) M1
For a subtraction of a suitable prob < 1, from 1

= 0.986 A1ft2
For correct answer, ft on their (i) and (ii)

**[7]**