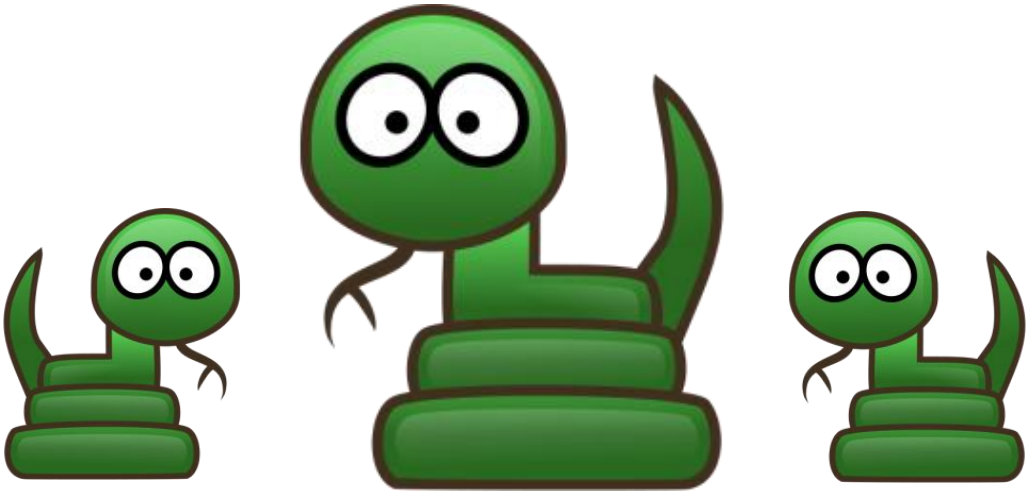
S1



REVISION PACK 3

SIMPLE PROBABILITY

1. Jamie is equally likely to attend or not to attend a training session before a football match. If he attends, he is certain to be chosen for the team which plays in the match. If he does not attend, there is a probability of 0.6 that he is chosen for the team.

(i) Find the probability that Jamie is chosen for the team.

[3]

(ii) Find the conditional probability that Jamie attended the training session, given that he was chosen for the team.

[3]

2. Two fair dice are thrown.

(i) Event A is 'the scores differ by 3 or more'. Find the probability of event A

[3]

(ii) Event B is 'the product of the scores is greater than 8'. Find the probability of event B.

[2]

(iii) State with a reason whether events A and B are mutually exclusive.

[2]

3. In country *A* 30% of people who drink tea have sugar in it. In country *B* 65% of people who drink tea have sugar in it. There are 3 million people in country *A* who drink tea and 12 million people in country *B* who drink tea. A person is chosen at random from these 15 million people.

(i) Find the probability that the person chosen is from country *A*

[1]

(ii) Find the probability that the person chosen does not have sugar in their tea.

[2]

(iii) Given that the person chosen does not have sugar in their tea, find the probability that the person is from country *B*.

[2]

4. Boxes of sweets contain toffees and chocolates. Box *A* contains 6 toffees and 4 chocolates, box *B* contains 5 toffees and 3 chocolates, and box *C* contains 3 toffees and 7 chocolates. One of the boxes is chosen at random and two sweets are taken out, one after the other, and eaten.

(i) Find the probability that they are both toffees.

[3]

(ii) Given that they are both toffees, find the probability that they both came from box *A*.

[3]

5. At a zoo, rides are offered on elephants, camels and jungle tractors. Ravi has money for only one ride. To decide which ride to choose, he tosses a fair coin twice. If he gets 2 heads he will go on the elephant ride, if he gets 2 tails he will go on the camel ride and if he gets 1 of each he will go on the jungle tractor ride.

(i) Find the probabilities that he goes on each of the three rides.

[2]

The probabilities that Ravi is frightened on each of the rides are as follows:

$$\text{elephant ride } \frac{6}{10}, \quad \text{camel ride } \frac{7}{10}, \quad \text{jungle tractor ride } \frac{8}{10}.$$

(ii) Draw a fully labelled tree diagram showing the rides that Ravi could take and whether or not he is frightened.

[2]

Ravi goes on a ride.

(iii) Find the probability that he is frightened.

[2]

(iv) Given that Ravi is **not** frightened, find the probability that he went on the camel ride.

[3]

6. A vegetable basket contains 12 peppers, of which 3 are red, 4 are green and 5 are yellow. Three peppers are taken, at random and without replacement, from the basket.

(i) Find the probability that the three peppers are all different colours.

[3]

(ii) Show that the probability that exactly 2 of the peppers taken are green is $\frac{12}{55}$.

[2]

(iii) The number of **green** peppers taken is denoted by the discrete random variable X. Draw up a probability distribution table for X.

[5]

7. There are three sets of traffic lights on Karinne's journey to work. The independent probabilities that Karinne has to stop at the first, second and third set of lights are 0.4, 0.8 and 0.3 respectively

(i) Draw a tree diagram to show this information.

[2]

(ii) Find the probability that Karinne has to stop at each of the first two sets of lights but does not have to stop at the third set.

[2]

(iii) Find the probability that Karinne has to stop at exactly two of the three sets of lights.

[3]

(iv) Find the probability that Karinne has to stop at the first set of lights, given that she has to stop at exactly two sets of lights.

[3]

8. Every day Eduardo tries to phone his friend. Every time he phones there is a 50% chance that his friend will answer. If his friend answers, Eduardo does not phone again on that day. If his friend does not answer, Eduardo tries again in a few minutes' time. If his friend has not answered after 4 attempts, Eduardo does not try again on that day.

(i) Draw a tree diagram to illustrate this situation.

[3]

(ii) Let X be the number of unanswered phone calls made by Eduardo on a day. Copy and complete the table showing the probability distribution of X.

x	0	1	2	3	4
$P(X = x)$		$\frac{1}{4}$			

[4]

(iii) Calculate the expected number of unanswered phone calls on a day.

[2]

9. A fair dice has four faces. One face is coloured pink, one is coloured orange, one is coloured green and one is coloured black. Five such dice are thrown and the number that fall on a green face are counted. The random variable X is the number of dice that fall on a green face.

(i) Show that the probability of 4 dice landing on a green face is 0.0146, correct to 4 decimal places.

[2]

(ii) Draw up a table for the probability distribution of X, giving your answers correct to 4 decimal places.

[5]

10. A fair die has one face numbered 1, one face numbered 3, two faces numbered 5 and two faces numbered 6.

(i) Find the probability of obtaining at least 7 odd numbers in 8 throws of the die.

[4]

The die is thrown twice. Let X be the sum of the two scores. The following table shows the possible values of X.

		Second throw					
		1	3	5	5	6	6
First throw	1	2	4	6	6	7	7
	3	4	6	8	8	9	9
	5	6	8	10	10	11	11
	6	5	8	10	10	11	11
	6	7	9	11	11	12	12
	6	7	9	11	11	12	12

(ii) Draw up a table showing the probability distribution of X.

[3]

(iii) Calculate E(X).

[2]

(iv) Find the probability that X is greater than E(X).

[2]

11. A die is biased so that the probability of throwing a 5 is 0.75 and the probabilities of throwing a 1, 2, 3, 4 or 6 are all equal.

(i) The die is thrown three times. Find the probability that the result is a 1 followed by a 5 followed by any even number.

[3]

(ii) Find the probability that, out of 10 throws of this die, at least 8 throws result in a 5.

[3]

(iii) The die is thrown 90 times. Using an appropriate approximation, find the probability that a 5 is thrown more than 60 times.

[5]

12. A box contains 300 discs of different colours. There are 100 pink discs, 100 blue discs and 100 orange discs. The discs of each colour are numbered from 0 to 99. Five discs are selected at random, one at a time, with replacement. Find

(i) the probability that no orange discs are selected,

[1]

(ii) the probability that exactly 2 discs with numbers ending in a 6 are selected,

[3]

(iii) the probability that exactly 2 orange discs with numbers ending in a 6 are selected,

[2]

(iv) the mean and variance of the number of pink discs selected.

[2]

13. The probability that New Year's Day is on a Saturday in a randomly chosen year is $\frac{1}{7}$.

(i) 15 years are chosen randomly. Find the probability that at least 3 of these years have New Year's Day on a Saturday.

[4]

(ii) 56 years are chosen randomly. Use a suitable approximation to find the probability that more than 7 of these years have New Year's Day on a Saturday.

[5]

14. Tyre pressures on a certain type of car independently follow a normal distribution with mean 1.9 bars and standard deviation 0.15 bars.

(i) Find the probability that all four tyres on a car of this type have pressures between 1.82 bars and 1.92 bars.

[5]

(ii) Safety regulations state that the pressures must be between $1.9 - b$ bars and $1.9 + b$ bars. It is known that 80% of tyres are within these safety limits. Find the safety limits.

[3]

SOLUTIONS

1. (i) $P(\text{team}) = 0.5 + 0.5 \times 0.6$ B1
One correct product

M1
Summing two 2-factor products

$= 0.8$ A13
Correct answer

(ii) $P(\text{training session} \mid \text{team}) = \dfrac{0.5}{0.5+0.5\times0.6}$ M1
Selecting correct term from (i) as their numerator

M1
Dividing by their (i) (must be < 1)

= 0.625 (5/8) A13
Correct answer

[6]

2. (i) list 14, 15, 16, 25, 26, 36,and reversed M1
For an attempt at listing

P (scores differ by 3 or more) = 12/36 A1
Selecting at least 6 correct pairs

(1/3)(0.333) A13
Correct answer

(ii) 20/36 M1
Some identification on the list, must include one of 25, 26, 33, 34, 35

A12
Correct answer

(iii) P ($A \cap B$) $\neq 0$ implies not mut excl, or equivalent B1
Correct statement about mut excl events

P ($A \cap B$) = 6/36 so not mut excl B1 ft2
Correct answer using their data

[7]

3. (i) P(A) = 0.2 B1 1
o.e. Must be single fraction or 20%

(ii) P(not S) = 0.2 × 0.7 + 0.8 × 0.35 M1
Summing two 2-factor probabilities or subtracting P(S) from 1

= 0.42 A12
o.e. Correct answer no decimals in fractions

(iii) P(B $|S'$) = $\dfrac{0.8 \times 0.35}{0.42}$ M1

$\dfrac{(1-\text{their}) \times 0.35}{\text{their}()}$ if marks lost in (i) or (ii)

= 0.667 A12
Correct answer c.w.o

[5]

4. (i) P(T, T) = $\frac{1}{3}\times\frac{6}{10}\times\frac{5}{9}\times\frac{1}{3}\times\frac{5}{8}\times\frac{4}{7}+\frac{1}{3}\times\frac{3}{10}\times\frac{2}{9}$ = 53/210 (0.252) B1

For one correct 3-factor term M1
For summing three 3-factor or 2-factor probs

A13
For correct answer

(ii) P(A TT) = 0.111/0.252 M1
For choosing only their P(A T T) in num or denom

For dividing by their (i) or what they think is P(T,T) M1

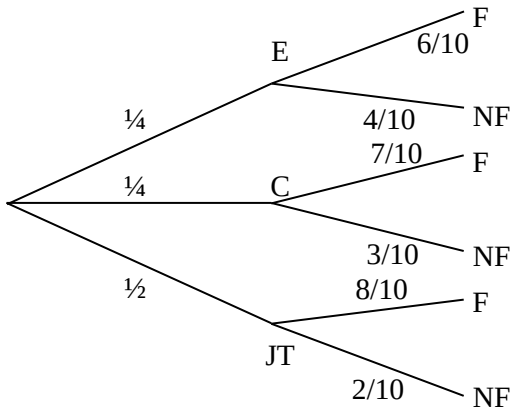
= 70/159 (0.440) A13
For correct answer using either 2 or 3-term probs
Constant prob B0M1A0M1M1A0 max

[6]

5. (i) P(E) = $\frac{1}{4}$, P(C) = $\frac{1}{4}$, P(JT) = $\frac{1}{2}$ B1
$\frac{1}{4}$, $\frac{1}{4}$, and $\frac{1}{2}$ seen oe

B12
3 evaluated probs correctly associated

(ii)



M1

E, C, JT then F on appropriate shape

A1ft2

All probs and labels showing and correct, ft their (i) if $\Sigma p = 1$.
If nothing seen in part (i) then give M1 A1ft bod provided their $\Sigma p = 1$
No retrospective marking

(iii) P(F) = (1/4 × 6/10) + (1/4 × 7/10) + (1/2 × 8/10) M1

Summing 3 appropriate two-factor products provided $\Sigma p = 1$

= 29/40 (0.725) B12

Correct answer

(iv) $P(C \mid NF) = \dfrac{P(C \cap NF)}{P(NF)}$ B1ft

1 – 29/40 seen in denom, ft 1 – their (iii)

$= \dfrac{3/40}{(1-29/40)}$ M1

attempt at cond prob with their $C \cap F$ or $C \cap NF$ in numerator

= 3/11 (0.273)

OR using ratios 3/(4 + 3 + 4) A13

correct answer

[9]

6. (i) P (all different) = $\frac{{}_3C_1 \times {}_4C_1 \times {}_5C_1}{{}_{12}C_3} =$ M1

Attempt using combinations, with ${}_{12}C_3$ denom, or P(*RGY*) in any order, i.e. 12 × 11 × 10 in denom

M1

Correct numerator, or multiplying by 6

= 3/11 (= 0.273) A13

Correct answer

(ii) *P*(exactly 2 *G*) = $\frac{{}_4C_2 \times {}_8C_1}{{}_{12}C_3}$ M1

Attempt using combinations, or mult any P ($GG\,\overline{G}$) × 3
Or P(*GGY*) × 3+P(*GGR*) × 3

= 12/55 *AG* A12

Correct answer *AG*

(iii)

x	0	1	2	3
P(*X* = *x*)	14/55	28/55	12/55	1/55
decimal	0.255	0.509	0.218	0.018

M1

For seeing P(0, 1, 2, 3) only and 1 or more probs

M1

For reasonable attempt at P(*X* = 0 or 1 or 3)

A1

For one correct probability seen other than P(*X* = 2)

A1

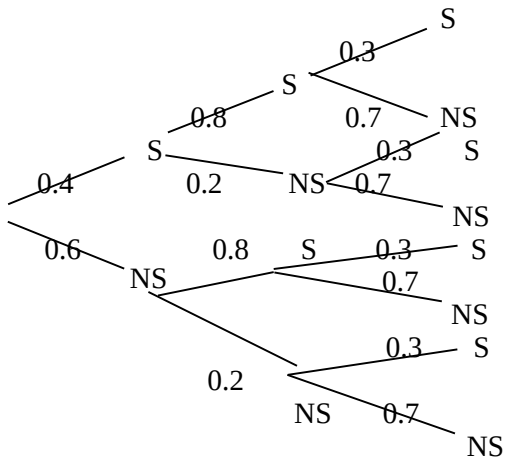
For a second probability correct other than P(*X* = 2)

A15

All correct

[10]

7. (i)



B1

Correct shape and labels

B12

Correct probabilities

(ii) P(*S, S, NS*) = 0.4 × 0.8 × 0.7 M1

Multiplying 3 probs once and 0.7 seen

= 0.224 (28/125) A12

Correct answer

(iii) P(*S, NS, S*) + P(*NS, S, S*) + 0.224 M1

Summing three different 3-factor terms

B1

Correct expression for P(*S, NS, S*) or P(*NS, S, S*)

= 0.392 (49/125) A13

Correct answer

(iv) P(stops at first light) | (stops at exactly 2 lights)

$= \text{P}\dfrac{(S, NS, S) \text{ or } (S, S, NS)}{0.392}$ M1

Summing two 3-factor terms in numerator (need not be different) (must be a division)

$= \dfrac{0.4\times0.2\times0.3 + 0.4\times0.8\times0.7}{0.392}$ M1* dep

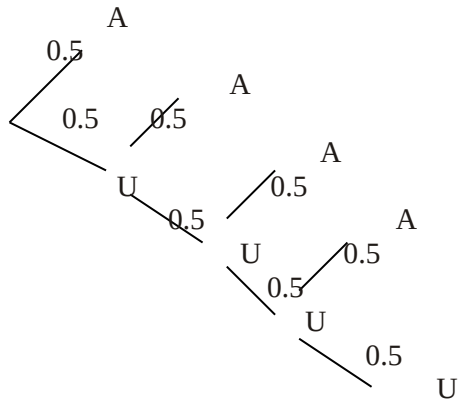
Dividing by their (iii) if their (iii) < 1, dep on previous M

= 0.633 (31/49) A1ft3

ft their E(*X*) provided 2 < E(*X*) < 12

[10]

8. (i)



M1

4 or 5 pairs A and U seen no extra bits but condone (0, 1) branches after any or all As.

A1

Exactly 4 pairs of A and U, must be labelled

A13

Correct diagram with all probs correct, allow A1ft for 4 correct pairs and (0,1) branch(es) or A1ft for 5 correct pairs and no (0, 1) branch(es)

(ii)

x	0	1	2	3	4
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{4}$	1/8	1/16	1/16

B1

P(0) correct

B1

P(2) correct

B1

P(3) correct

B14

P(4) correct

(iii) $E(X) = 15/16$ (0.938 or 0.9375)
attempt at $\Sigma(xp)$ only with no other numbers

M1

A12

correct answer

[9]

9. (i) $P(G, G, G, G, NG) = (0.25)^4 \times (0.75)^1 \times {}_5C_4$ M1

For relevant binomial calculation, need ${}_5C_r$ or 5 or all 5 options

$= 0.0146$ AG A12

For correct answer. AG

(ii)

X	0	1	2
$P(X = x)$	0.2373	0.3955	0.2637

B1

For all correct X values

B1

For one correct prob excluding $P(X = 4)$

B1

For 2 correct probs excluding $P(X = 4)$

X	3	4	5
$P(X = x)$	0.0879	0.0146	0.0010

B1

For 3 correct probs excluding $P(X = 4)$

B15

All correct and in decimals

[7]

10. (i) P(odd) = 2/3 or 0.667 B1

Can be implied if normal approx used with $\mu = 5.333 (= 8 \times 2/3)$

$P(7) = {}_8C_7 (2 / 3)^7 (1/ 3)$

$= 0.156$ M1

Binomial expression with C in and 2/3 and 1/3 in powers summing to 8

$P(8) = (2/3)^8 = 0.0390$ M1

Summing P(7) + P(8) binomial expressions

P(7 or 8) = 0.195 (1280/6561) A14

Correct answer

(ii)

x	2	4	6	7	8
P($X = x$)	1/36	2/36	5/36	4/36	4/36

B1

Values of x all correct in table of probabilities

x	9	10	11	12
P($X = x$)	4/36	4/36	8/36	4/36

B23

All probs correct and not duplicated, –1 ee

(iii) $E(X) = \sum p_i x_i$

$= 2 \times 1/36 + 4 \times 2/36 + ...$ M1

attempt to find $\sum p_i x_i$, all $p < 1$ and no further division of any sort

$= 312/36$ (26/3) (8.67) A12

correct answer

(iv) $P(X > E(X)) = P(X = 9, 10, 11, 12)$ M1

attempt to add their relevant probs

$= 20/36$ (5/9) (0.556) A12

correct answer

[11]

11. (i) $(0.05)(0.75)(0.15)$ M1

Multiplying 3 probs only, no Cs

$= 0.00563$ (9 / 1600) B1

0.05 or 0.15 or 1/5 × $\frac{1}{4}$ seen

A13

Correct answer

(ii) P(at least 8) = P(8, 9, 10) B1

Binomial expression involving $(0.75)^r(0.25)^{10-r}$ and a C, $r \neq 0$ or 10

$= {}_{10}C_8(0.75)^8(0.25)^2 + {}_{10}C_9(0.75)^9(0.25) + (0.75)^{10}$ M1

Correct unsimplified expression can be implied

$= 0.526$ A13

Correct answer

(iii) $\mu = 90 \times 0.75 = 67.5$
$\sigma^2 = 90 \times 0.75 \times 0.25 = 16.875$ B1
90×0.75 (67.5) and
$90 \times 0.75 \times 0.25$ (16.875 or 16.9) seen

$P(X > 60)$ M1
For standardising, with or without cc,
must have $\sqrt{\ }$ on denom

$= 1 - \Phi\left(\frac{60.5 - 67.5}{\sqrt{(16.875)}}\right) = \Phi(1.704)$ M1
For use of continuity correction 60.5 or
59.5

M1
For finding an area > 0.5 from their z

$= 0.956$ A15
For answer rounding to 0.956

[11]

12. (i) P(no orange) = $(2/3)^5$ or 0.132 or 32/243 B1 1
For correct final answer either as a decimal or a fraction

(ii) P(2 end in 6) = $(1/10)^2 \times (9/10)^3 \times {}_5C_2$ B1
For using $(1/10)^k$ $k > 1$

M1
For using a binomial expression with their 1/10
or seeing some $p^2 * (1 - p)^3$

$= 0.0729$ A1 3
For correct answer

(iii) P(2 orange end in 6) = $(1/30)^2 \times (29/30)^3 \times {}_5C_2$ M1
For their (1/10)/3 seen

$= 0.0100$ accept 0.01 A12
For correct answer

(iv) $n = 5$, $p = 1/3$, B1
For recognising $n = 5$, $p = 1/3$

mean = 5/3, variance = 10/9 B1 ft2
For correct mean and variance, ft their n and p, $p < 1$

[8]

13. (i) $P(\geq 3) = 1 - P(0, 1, 2)$ M1
For attempt at 1 – P(0, 1, 2) or 1 – P(0, 1, 2, 3) or P(3...15) or P(4...15)

$= 1 - (6/7)^{15} - {}_{15}C_1\,(1/7)\,(6/7)^{14} - {}_{15}C_2\,(1/7)^2\,(6/7)^{13}$ M1
For 1 or more terms with 1/7 and 6/7 to powers which sum to 15 and ${}_{15}C_{\text{something}}$

(= 1 – 0.0990 – 0.2476 – 0.2889) A1
Completely correct unsimplified form

= 0.365 (accept 0.364) A14
Correct final answer

(ii) $\mu = 56 \times 1/7$ (= 8)
$\sigma^2 = 56 \times 1/7 \times 6/7$ (= 6.857) B1
8 and 6.857 or 6.86 or 2.618 seen or implied

P(more than 7) = $1 - \Phi\left(\frac{7.5-8}{\sqrt{6.857}}\right)$ M1
Standardising attempt with or without cc, must have square root

$= \Phi\left(\frac{8-7.5}{\sqrt{6.857}}\right) = \Phi(0.1909)$ M1
Continuity correction either 7.5 or 6.5

M1
Final answer > 0.5 (award this if the long way is used and the final answer is > 0.5)

= 0.576 A15
Correct final answer

[9]

14. (i) $z_1 = 0.02/0.15 = 0.1333$ M1
For standardising one value, no cc

$z_2 = -0.08/0.15 = -0.5333$ M1
For standardising the other value, no cc. SR ft on no sq rt

area= $\Phi(0.1333) - \Phi(-0.533)$
$= \Phi(0.1333) - [1 - \Phi(0.5333)]$
$= 0.5529 + 0.7029 - 1$ M1
For finding correct area (i.e. two Φs – 1)

= 0.256 A1
For correct answer

Prob all 4 = $(0.256)^4$ (0.00428 to 0.00430) A1ft5
For correct answer, ft from their (i), if $p<1$, allow 0.0043

(ii) $z = \pm 1.282$ or 1.28 or 1.281 B1
For correct z, + or – or both

$\pm 1.282 \quad \frac{b}{0.15}$ M1
For seeing an equation involving + or – of their z, b, 0.15 (their z can only be 0.842 or 0.84 or 0.841)

limits between 1.71 and 2.09 A1ft3
both limits needed, ft 1.77 to 2.03 on 0.842 only

[8]