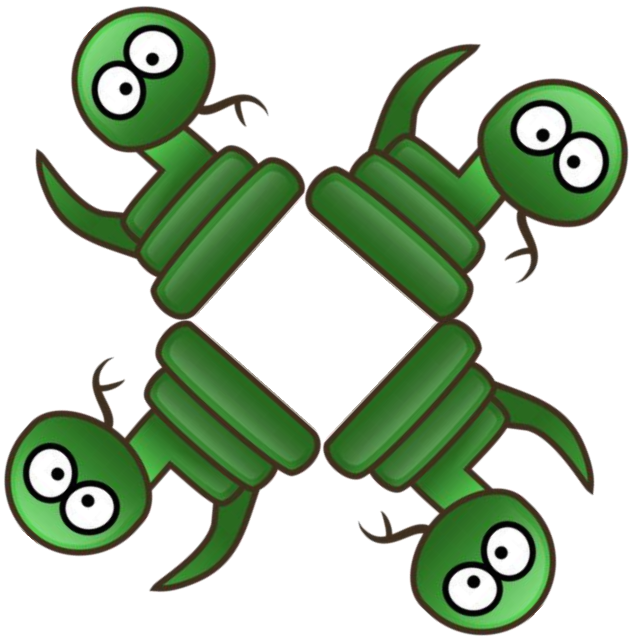
S1



REVISION PACK 4

PROBABILITY DISTRIBUTIONS

1. Gohan throws a fair tetrahedral die with faces numbered 1, 2, 3, 4. If she throws an even number then her score is the number thrown. If she throws an odd number then she throws again and her score is the sum of both numbers thrown. Let the random variable X denote Gohan's score.

(i) Show that $P(X = 2) = \frac{5}{16}$.

[2]

(ii) The table below shows the probability distribution of X.

x	2	3	4	5	6	7
$P(X = x)$	$\frac{5}{16}$	$\frac{1}{16}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

Calculate $E(X)$ and $Var(X)$.

[4]

2. 32 teams enter for a knockout competition, in which each match results in one team winning and the other team losing. After each match the winning team goes on to the next round, and the losing team takes no further part in the competition. Thus 16 teams play in the second round, 8 teams play in the third round, and so on, until 2 teams play in the final round.

(i) How many teams play in only 1 match? **[1]**

(ii) How many teams play in exactly 2 matches? **[1]**

(iii) Draw up a frequency table for the numbers of matches which the teams play. **[3]**

(iv) Calculate the mean and variance of the numbers of matches which the teams play. **[4]**

3. The discrete random variable X has the following probability distribution.

x	0	1	2	3	4
$P(X = x)$	0.26	q	$3q$	0.05	0.09

(i) Find the value of q. **[2]**

(ii) Find $E(X)$ and $Var(X)$. **[3]**

4. In a competition, people pay \$1 to throw a ball at a target. If they hit the target on the first throw they receive \$5. If they hit it on the second or third throw they receive \$3, and if they hit it on the fourth or fifth throw they receive \$1. People stop throwing after the first hit, or after 5 throws if no hit is made. Mario has a constant probability of $\frac{1}{5}$ of hitting the target on any throw, independently of the results of other throws.

(i) Mario misses with his first and second throws and hits the target with his third throw. State how much profit he has made.

[1]

(ii) Show that the probability that Mario's profit is \$0 is 0.184, correct to 3 significant figures.

[2]

(iii) Draw up a probability distribution table for Mario's profit.

[3]

(iv) Calculate his expected profit.

[2]

5. The random variable X takes the values –2, 0 and 4 only. It is given that $P(X = -2) = 2p$, $P(X = 0) = p$ and $P(X = 4) = 3p$.

(i) Find p. **[2]**

(ii) Find $E(X)$ and $Var(X)$. **[4]**

6. It is known that, on average, 2 people in 5 in a certain country are overweight. A random sample of 400 people is chosen. Using a suitable approximation, find the probability that fewer than 165 people in the sample are overweight.

[5]

7. (i) The daily minimum temperature in degrees Celsius (°C) in January in Ottawa is a random variable with distribution N (–15.1, 62.0). Find the probability that a randomly chosen day in January in Ottawa has a minimum temperature above 0°C.

[3]

(ii) In another city the daily minimum temperature in °C in January is a random variable with distribution N (μ, 40.0). In this city the probability that a randomly chosen day in January has a minimum temperature above 0°C is 0.8888. Find the value of μ.

[3]

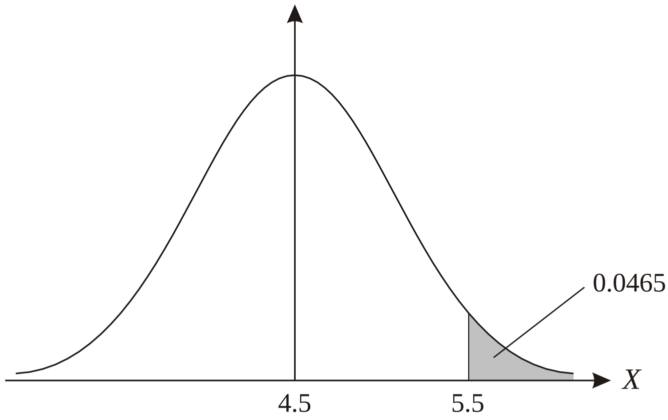
8. (a) The random variable X is normally distributed. The mean is twice the standard deviation. It is given that $P(X > 5.2) = 0.9$. Find the standard deviation.

[4]

(b) A normal distribution has mean μ and standard deviation σ. If 800 observations are taken from this distribution, how many would you expect to be between $\mu - \sigma$ and $\mu + \sigma$?

[3]

9.



The random variable X has a normal distribution with mean 4.5. It is given that $P(X > 5.5) = 0.0465$ (see diagram).

(i) Find the standard deviation of X.

[3]

(ii) Find the probability that a random observation of X lies between 3.8 and 4.8.

[4]

10. (i) Give an example of a variable in real life which could be modelled by a normal distribution.

[1]

(ii) The random variable X is normally distributed with mean μ and variance 21.0. Given that $P(X > 10.0) = 0.7389$, find the value of μ.

[3]

(iii) If 300 observations are taken at random from the distribution in part (ii), estimate how many of these would be greater than 22.0.

[4]

11. In a certain country the time taken for a common infection to clear up is normally distributed with mean μ days and standard deviation 2.6 days. 25% of these infections clear up in less than 7 days.

(i) Find the value of μ. **[4]**

In another country the standard deviation of the time taken for the infection to clear up is the same as in part (i), but the mean is 6.5 days. The time taken is normally distributed.

(ii) Find the probability that, in a randomly chosen case from this country, the infection takes longer than 6.2 days to clear up. **[3]**

12. The lengths of fish of a certain type have a normal distribution with mean 38 cm. It is found that 5% of the fish are longer than 50 cm.

(i) Find the standard deviation. **[3]**

(ii) When fish are chosen for sale, those shorter than 30 cm are rejected. Find the proportion of fish rejected. **[3]**

(iii) 9 fish are chosen at random. Find the probability that at least one of them is longer than 50 cm. **[2]**

13. The volume of milk in millilitres in cartons is normally distributed with mean μ and standard deviation 8. Measurements were taken of the volume in 900 of these cartons and it was found that 225 of them contained more than 1002 millilitres.

(i) Calculate the value of μ.

[3]

(ii) Three of these 900 cartons are chosen at random. Calculate the probability that exactly 2 of them contain more than 1002 millilitres.

[2]

14. A survey of adults in a certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist and 9% did not wear a watch.

(i) A random sample of 14 adults was taken. Find the probability that more than 2 adults did not wear a watch.

[4]

(ii) A random sample of 200 adults was taken. Using a suitable approximation, find the probability that more than 155 wore a watch on their left wrist.

[5]

15. On a certain road 20% of the vehicles are trucks, 16% are buses and the remainder are cars.

(i) A random sample of 11 vehicles is taken. Find the probability that fewer than 3 are buses. **[3]**

(ii) A random sample of 125 vehicles is now taken. Using a suitable approximation, find the probability that more than 73 are cars. **[5]**

16. In tests on a new type of light bulb it was found that the time they lasted followed a normal distribution with standard deviation 40.6 hours. 10% lasted longer than 5130 hours.

(i) Find the mean lifetime, giving your answer to the nearest hour. **[3]**

(ii) Find the probability that a light bulb fails to last for 5000 hours. **[3]**

(iii) A hospital buys 600 of these light bulbs. Using a suitable approximation, find the probability that fewer than 65 light bulbs will last longer than 5130 hours. **[4]**

17. On any occasion when a particular gymnast performs a certain routine, the probability that she will perform it correctly is 0.65, independently of all other occasions.

(i) Find the probability that she will perform the routine correctly on exactly 5 occasions out of 7.

[2]

(ii) On one day she performs the routine 50 times. Use a suitable approximation to estimate the probability that she will perform the routine correctly on fewer than 29 occasions.

[5]

(iii) On another day she performs the routine n times. Find the smallest value of n for which the expected number of correct performances is at least 8.

[2]

18. A manufacturer makes two sizes of elastic bands: large and small. 40% of the bands produced are large bands and 60% are small bands. Assuming that each pack of these elastic bands contains a random selection, calculate the probability that, in a pack containing 20 bands, there are

(i) equal numbers of large and small bands, **[2]**

(ii) more than 17 small bands. **[3]**

An office pack contains 150 elastic bands.

(iii) Using a suitable approximation, calculate the probability that the number of small bands in the office pack is between 88 and 97 inclusive. **[6]**

19. On a production line making toys, the probability of any toy being faulty is 0.08. A random sample of 200 toys is checked. Use a suitable approximation to find the probability that there are at least 15 faulty toys.

[5]

SOLUTIONS

1. (i) $P(X = 2) = 1/4 \times 1/4 + 1/4 = 5/16$ AG — M1

Considering cases (1, 1) and (2)

OR can use a table

	1	2	3	4
1	2	2	4	4
2	3	2	5	4
3	4	2	6	4
4	5	2	7	4

A1 2

Correct given answer legitimately obtained
(1/16 + 4/16 needs some justification but 1/16 + 1/4 is acceptable)

(ii) $E(X) = \Sigma xp$ — M1

Using correct formula for $E(X)$, no extra division

$= 15/4$ (3.75) — A1

Correct answer

$\text{Var}(X) = 2^2 \times 5/16 + 3^2 \times 1/16 + 4^2 \times 3/8 + ... - (15/4)^2$ — M1

Using a variance formula correctly with mean^2 subtracted numerically, no extra division

$= 260/16 - 225/16 = 35/16$ (2.19) — A1 4

Correct final answer

[6]

2. (i) 16 — B1 1

(ii) 8 — B1 1

(iii) Matches 1,2,3,4,5 — M1

Matches	1	2	3	4	5
freq	16	8	4	3	2

A1

3 correct frequencies

A1 3

All correct

(iv) mean = 62/32 — M1

Using their $\Sigma fx/\Sigma f$

$= 1.9375$ (= 1.94) — A1

Correct answer

$\text{var} = 166/32 - (62/32)^2$ — M1

Subst in $\Sigma fx^2 - (\Sigma fx/n)^2$ formula

$= 1.43$ — A1 4

Correct answer, or B2 if used calculator

[9]

3. (i) $q + 3q + 0.26 + 0.05 + 0.09 = 1$ M1
Equation with q in summing probs to 1 must be probs

$q = 0.15$ A12
Correct answer

(ii) $E(X) = 1.56$ B1ft
Correct final answer, ft on wrong q

$\text{Var}(X) = 0.15 + 1.8 + 0.45 + 1.44 - \text{mean}^2$ M1
Subst in $\Sigma px^2 - \text{mean}^2$ formula

$= 1.41$ A13
Correct final answer

[5]

4. (i) \$2 B11
For correct answer

(ii) P(MMMH) +P(MMMMH) M1
For attempting to sum P(MMMH) and P(MMMMH)

$= 0.8^3 \times 0.2 + 0.8^4 \times 0.2 = 0.184$ **AG** A12
For correct answer

(iii)

x	4	2	0	–1
$P(X = x)$	0.2	0.288	0.184	0.328

For one correct prob other than 0.184 B1

For another correct prob other than 0.184, ft only if the –1 ignored and their 3rd prob is 1 – Σ the other 2 B1ft

For correct table, can have separate 2s B13

(iv) $E(X) = 0.8 + 0.576 - 0.328$ M1
For attempt at $\Sigma\, xp$ from their table, at least 2 non-zero terms

= \$1.05 A12
For correct answer

[8]

5. (i) $2p + p + 3p = 1$ M1
Equation involving ps and summing to 1

$p = 1/6$ (= 0.167) A12
Correct answer

(ii) $E(X) = -2 \times 2/6 + 0 + 4 \times 3/6$ M1

Using correct formula for $E(X)$, in terms of p or their $p < 1$

$= 4/3\ (= 1.33)$ A1ft

Correct expectation ft on their p if $p \le 1/3$

$\text{Var}(X) = 4 \times 2/6 + 0 + 16 \times 3/6 - (4/3)^2$ M1

Substitution in their Σpx^2 – their $E^2(X)$ need 2 terms

$= 7.56\ (68/9)$ A14

Correct answer

[6]

6. $\mu = 160$, $\sigma^2 = 96$ B1

For 160 and 96 seen or implied by 9.798

$P(\le 165) = \Phi\left(\frac{164.5 - 160}{\sqrt{96}}\right) = \Phi(0.4593)$ M1

For standardising, must have square root

M1

For continuity correction, either 165.5 or 164.5

M1

For using tables and finding correct area (i.e.> 0.5)

$= 0.677$ A1

For correct answer

[5]

7. (i) $P(X > 0) = 1 - \Phi\ \frac{0 - 15.1}{\sqrt{62}}$ M1

Standardising, sq rt, no cc

$= 1 - \Phi\ (1.918)$

$= 1 - 0.9724$ M1

Prob < 0.5 after use of normal tables

$= 0.0276$ or answer rounding to A13

Correct answer

(ii) $z = -1.22$ B1

$z = \pm 1.22$

$-1.22 = \frac{0 - \mu}{\sqrt{40}}$ M1

an equation in μ, recognisable z, $\sqrt{40}$, no cc

$\mu = 7.72$ c.a.o A13

correct answer c.w.o from same sign on both sides

[6]

8. (a) $\frac{5.2-2s}{s} = -1.282$ M1

Equation with ± correct LHS seen here or later, can be μ or s, no cc

B1

±1.282 seen accept ± 1.28 or anything in between

M1

solving their equation with recognisable z-value and only 1 unknown occurring twice

s = 7.24 or 7.23 A14

correct final answer

(b) $\Phi\left(\frac{\mu+\sigma-\mu}{\sigma}\right) = 0.8413$ B1

0.8413 (p) seen or implied (can use their own numbers)

P($|z|$ < 1) = 0.3413 × 2 = 0.6826 M1

finding the correct area i.e. 2p – 1

0.6826 ×800 = 546 (accept 547) A13

correct answer, must be a positive integer

OR

SR 800 × 2/3 = 533 or 534 SR B1

for 2/3

B1

for 533 or 534 or B2 if 533 or 534 and no working

[7]

9. (i) z = ± 1.68 B1

Number rounding to 1.68 seen

$z = \frac{5.5-4.5}{\sigma}$ M1

Standardising and attempting to solve with their z, ; must be z value, no cc, no σ^2, no $\sqrt{\sigma}$

σ = 0.595 accept 25/42 A13

Correct answer

(ii) $z_1 = \frac{3.8-4.5}{0.5952} = -1.176$ M1

For standardising 3.8 or 4.8, mean 4.5 not 5.5, their σ or $\sqrt{\sigma}$ or σ^2 in denom

$z_2 = \dfrac{4.8-4.5}{0.5952}$ – 0.504 A1ft

One correct z-value, ft on their σ

prob = Φ(0.504) – (1 – Φ(1.176))
0.6929 – (1 – 0.8802) M1

Correct area ie $\Phi_1 + \Phi_2 - 1$ or $\Phi_1 - \Phi_2$ if μ taken to be 5.5

= 0.573 A14

Correct answer only

[7]

10. (i) heights, weights, times etc of something B11

Any sensible set of data, must be qualified

(ii) $z = 0.64 = \dfrac{\mu - 10}{\sqrt{21}}$ B1

$z = \pm\ 0.64$ seen

M1

equation relating 10, $\sqrt{21}$, 21, μ and their z or 1 – their z, (must be a recognisable z value ie not 0.77)

$\mu = 12.9$ A13

correct answer

(iii) $z = \dfrac{22-12.9}{\sqrt{21}}$

= 1.986 M1

standardising, with or without sq rt, no cc, must be their mean

P(X > 22) = 1 – Φ(1.986)
= 1 – 0.9765
= 0.0235 M1ft

correct area ie < 0.5, ft on their mean >22

300 × 0.0235 = 7.05 M1

mult by 300

answer = 7 A14

correct answer, accept 7 or 8 must be integer

[8]

11. (i) $-0.674 = \dfrac{7-\mu}{2.6}$ B1

± 0.674 seen only

M1

Standardising must have a recognisable z-value, no cc and 2.6

M1

For solving their equation with recognisable z-value, μ and 2.6 not

1 – 0.674 or 0.326, allow cc

$\mu = 8.75$ A14
Correct answer

(ii) $P(X > 6.2) = P\left(z > \frac{6.2-6.5}{2.6}\right)$ M1
Standardising, no cc on the 6.2

prob > 0.5 M1
= P(z > –0.1154)
= 0.546 A13
Correct answer

[7]

12. (i) $1.645 = \frac{50-38}{\sigma}$ B1
Using z = +/– 1.645 or 1.65

M1
Equation with 38, 5, σ and a recognizable z-value

$\sigma = 7.29$ A13
Correct answer

(ii) $z = \frac{30-38}{their\,\sigma}$ (= –1.097) M1
Standardising, no cc

P(z < 30) = 1 – Φ(1.097) M1
Finding correct area ie < 0.5

= 1 – 0.8637
= 0.136 A13
Correct answer

(iii) $1 - (0.95)^9$ B1
$(0.95)^9$ seen

– 0.370 B12
correct answer

[8]

13. (i) $z = 0.674$ B1
± 0.674 or rounding to, seen, e.g. 0.6743

$\frac{1002 - \mu}{8} = 0.674$ M1
Standardising and attempting to solve for μ, must use recognisable z-value, no cc, no sq rt, no sq

$\mu = 997$ A13
Correct answer rounding to 997

(ii) P(2) = $3\times\frac{225}{900}\times\frac{224}{899}\times\frac{675}{898}$ M1

900 × 899 × 898 or $^{900}C_3$ seen in denom

= 0.140

or $\dfrac{^{225}C_2\times{}^{675}C_1}{^{900}C_3}$ A1 2

Correct answer not 0.141 or 0.14

14. (i) 1 – P(0, 1, 2) M1

for 1 – P(0, 1, 2)

$= 1 - ((0.91)^{14} + (0.09)(0.91)^{13} \times {}_{14}C_1$
$+ (0.09)^2(0.91)^{12} \times {}_{14}C_2)$ B1

Correct numerical expression for P(0) or P(1)

B1

Correct numerical expression for P(2)

= 1 – (0.2670 + 0.3698 + 0.2377)
= 0.126 A1 4
Correct answer

(ii) $\mu = 200 \times 0.76 = 152$, B1

For both mean and variance correct

$\sigma^2 = 200 \times 0.76 \times 0.24 = 36.48$

P(X > 155) M1

For standardizing, with or without cc, must have $\sqrt{}$ on denom

$= 1 - \Phi\left(\dfrac{155.5-152}{\sqrt{36.48}}\right) = 1 - \Phi(1.5795)$ M1

For use of continuity correction 154.5 or 155.5

M1

For finding an area < 0.5 for their z

= 1 – 0.7188 = 0.281 A1 5

For answer rounding to 0.281

[9]

15. (i) P(X < 3) = P(0) + P(1) + P(2) M1

Binomial term with $^{11}C_r\, p^r\,(1-p)^{11-r}$ seen

$= (0.84)^{11} + (0.16)(0.84)^{10} \times {}^{11}C_1 +$
$(0.16)^2(0.84)^9 \times {}^{11}C_2$ M1

Correct expression for P(0, 1, 2) or P(0, 1, 2, 3)
Can have wrong p

= 0.1469 + 0.30782 + 0.2931
= 0.748 A1 3
Correct final answer. Normal approx M0 M0 A0

	(ii)	$\mu = 125 \times 0.64 = 80$ $\sigma^2 = 125 \times 0.64 \times 0.36 = 28.8$ 80 and 28.8 or 5.37 seen	B1
		$P(X > 73) = 1 - \Phi\left(\dfrac{73.5-80}{\sqrt{28.8}}\right)$ standardising, with or without cc, must have sq rt in denom	M1
		continuity correction 73.5 or 72.5 only	M1
		$= \Phi\ (1.211)$ correct region (> 0.5 if mean > 73.5, vv if mean < 73.5)	M1
		= 0.887 correct answer	A15
			[8]
16.	(i)	$1.282 = (5130 - \mu)/40.6$ For ± 1.282 seen, or 1.28, 1.281, not 1.29 or 1.30	B1
		For standardising, with or without sq rt, squared, no cc	M1
		μ = 5080 (5078) rounding to 5080 For correct answer	A13
	(ii)	P(<5000) = Φ[(5000 – 5078)/ 40.6] For standardising, criteria as above, can include cc	M1
		= Φ(–1.921) For correct area found using tables ie < 0.5ft on wrong (i)	M1
		= 1 – 0.9727 = 0.0273 or 2.73% For correct answer, accept 0.0274	A13
	(iii)	μ = 60, var = 54 For 60 and 54 seen (could be sd or variance)	B1
		P(fewer than 65) = Φ (64.5 – 60) / √ 54 For using 64.5 or 65.5 in a standardising process	M1
		= Φ(0.6123) For standardising, must have √(their 54) in denom	M1
		= 0.730 accept 0.73 For correct answer	A14
			[10]

17.	(i)	$P(X = 5) = (0.65)^5 \times (0.35)^2 \times {}_7C_5$ Expression with 3 terms, powers summing to 7 and *a* ${}_7C$ term	M1
		= 0.298 allow 0.2985 Correct answer	A12
	(ii)	$\mu = 50 \times 0.65$ (= 32.5), $\sigma^2 = 50 \times 0.65 \times 0.35$ (= 11.375) 32.5 and 11.375 seen or implied	B1
		P(fewer than 29) = $\Phi\left(\frac{28.5-32.5}{\sqrt{11.375}}\right)$ standardising, with or without cc, must have sq rt	M1
		$= 1 - \Phi(1.186)$ for continuity correction 28.5 or 29.5	M1
		$= 1 - 0.8822$ correct area ie < 0.5 must be from a normal approx	M1
		= 0.118 correct answer	A15
	(iii)	$0.65\,n \geq 8$ equality or inequality with *np* and 8	M1
		smallest $n = 13$ correct answer	A12
			[9]
18.	(i)	$(0.6)^{10} \times (0.4)^{10} \times {}_{20}C_{10}$ 3 term binomial expression involving ${}_{20}C_{\text{something}}$ and powers summing to 20	M1
		Correct final answer	A12

(ii) P(18, 19, 20) M1
Summing three or 4 binomial expressions

$= (0.6)^{18} (0.4)^2 \ {}_{20}C_2 + (0.6)^{19}(0.4)^1 \ {}_{21}C_1$
$+(0.6)^{20}$
= 0.003087 + 0.000487 + 0.00003635 A1
One correct unsimplified expression allow 0.4 0.6 muddle

= 0.00361 A1
Correct answer

OR:

using normal approx N(12,4.8) M1
Standardising, cc 16.5 or 17.5, their mean, $\sqrt{\ }$ (their var)

$$z = \frac{17.5 - 12}{\sqrt{4.8}}$$
= 2.52 A1
2.51 seen

Prob = 1 – 0.9940 = 0.0060 A13
0.0060 seen must be 0.0060

(iii) $\mu = 150 \times 0.60 = 90$
$\sigma^2 = 150 \times 0.60 \times 0.40 = 36$ B1
For seeing 90 and 36

P(88 < X < 97)
$= \Phi\left(\frac{97.5-90}{6}\right) - \Phi\left(\frac{87.5-90}{6}\right)$ M1
For standardising , with or without cc, must have sq rt on denom

= Φ(1.25) – Φ(–0.4166) M1
one continuity correction 97.5 or 96.5 or 87.5 or 88.5

= 0.8944 – (1 – 0.6616) A1
0.8944 or 0.6616 or 0.3384 or 0.3944 or 0.1616 seen

= 0.556 M1
subtracting a probability from their standardised 97 prob

correct answer A16
[11]

19. mean = 200 × 0.08 = 16
var = 14.72 B1
For both 16 and 14.7 seen

$P(X \geq 15) = 1 - \Phi\left(\frac{14.5-16}{\sqrt{14.72}}\right)$ M1
For standardising, with or without cc, must have

$\sqrt{\ }$ in denom

= Φ (0.391) M1

For use of continuity correction 14.5 or 15.5

= 0.652 M1

For finding a prob > 0.5 from their *z*, legit

A1

For answer rounding to 0.652 c.w.o

[5]